CHAPTER

06

Electromagnetic induction

The phenomenon of generating current/emf in a conducting circuit by change in strength, position or orientation of an associated external magnetic field is called **electromagnetic induction**. The emf, so developed is called **induced emf**. If the conductor is in the form of a closed circuit, a current flows in the circuit. This is called **induced current**. It is the reverse process of magnetic field produced by electric current. EMI was discovered by Michael Faraday in 1831. In this chapter, we will study about the Lenz's law, motional emf and inductances.

Inside

1) Magnetic flux
- Faraday's laws of electromagnetic induction
- Lenz's law and conservation of energy

2) Motional electromotive force
- General form of motional emf
- Fleming's right hand thumb rule to find the direction of induced current
- Induction of field

3) Self-induction
- Kirchhoff's second law with an inductor

4) Mutual induction
- Mutual inductance for two concentric coils
- Mutual inductance of two long co-axial solenoids
- Mutual inductance of a solenoid surrounded by a coil
- Coefficient of coupling
- Combination of mutual inductances
- Growth and decay of current in *L-R* circuit

1 Magnetic flux

The total number of magnetic field lines crossing through any surface, normally when it is placed in a magnetic field is known as the magnetic flux of that surface.

Consider an element of area $d\mathbf{S}$ on an arbitrary shaped surface as shown in Fig. 6.1. If the magnetic field at this element is **B**, then the magnetic flux through the element is

$$d\phi_B = \mathbf{B} \cdot d\mathbf{S} \Rightarrow d\phi_B = BdS\cos\theta$$

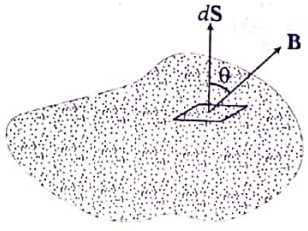


Fig. 6.1

Here, $d\mathbf{S}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dS and θ is the angle between **B** and $d\mathbf{S}$ at that element. In general, $d\phi_B$ varies from element to element.

The total magnetic flux through the surface is the sum of the contributions from the individual area elements.

$\therefore$

$$\phi_B = \int BdS\cos\theta = \int \mathbf{B} \cdot d\mathbf{S}$$

Note

(i) As magnetic lines of force are closed curves (*i.e.*, monopoles do not exist), total magnetic flux linked with a closed surface is always zero.

(ii) A special case, in which **B** is uniform over a plane surface with total area *S*, magnetic flux is given by $\phi_B = BS\cos\theta$

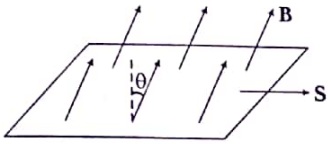
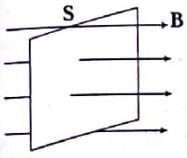
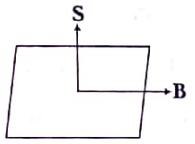
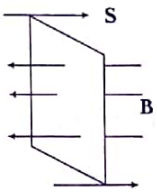


Fig. 6.2 A plane of surface area S placed on uniform field B

Specific conditions

For $\theta = 0°$	For $\theta = 90°$	For $\theta = 180°$
		
B is parallel to surface vector *i.e.*, **S**.	**B** is perpendicular to surface vector *i.e.*, **S**	**B** is antiparallel to surface vector, *i.e.*, **S**.
$\Rightarrow \phi_B = BS\cos 0°$	$\Rightarrow \phi_B = BS\cos 90°$	$\Rightarrow \phi_B = BS\cos 180°$
$= BS$	$= 0$	$= BS(-1) = -BS$
$\Rightarrow \phi_B$ = maximum		

Units and dimension of magnetic flux

(i) Magnetic flux is a scalar quantity.

(ii) The SI unit of magnetic flux is **tesla-metre**^{2} (1 T-m^2). This unit is called **weber (Wb)**.

$$1\ \text{Wb} = 1\ \text{T-m}^2 = 1\ \text{N-m/A}$$

The CGS unit of flux is maxwell (Mx).

$$1\text{Mx} = 1\ \text{gauss-cm}^2$$

$$1\ \text{Wb} = 1\ \text{T} \times 1\ \text{m}^2 = 10^4\ \text{G} \times 10^4\ \text{cm}^2$$

or $1\ \text{Wb} = 10^8\ \text{Mx}$

Thus, unit of magnetic field is also weber/m^2 (1Wb/m^2)

or $1\ \text{T} = 1\ \text{Wb/m}^2$

(iii) The dimensional formula of magnetic flux is $[ML^2T^{-2}A^{-1}]$.

Example 6.1 *A uniform magnetic field exists in the space* $\mathbf{B} = B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} - B_3\hat{\mathbf{k}}$. *Find the magnetic flux through an area* **S**, *if the area* **S** *is in yz-plane.*

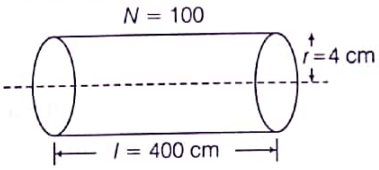
Sol. Since, the field is uniform, we can use formula, $\phi_B = \mathbf{B}\cdot\mathbf{S}$

Area **S** is in *yz*-plane, it means $\mathbf{S} = S\hat{\mathbf{i}}$

Hence, $\phi_B = (B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} - B_3\hat{\mathbf{k}})\cdot(S\hat{\mathbf{i}}) = B_2S.$

Example 6.2 *A long solenoid of radius 4 cm, length 400 cm carries a current of 3 A. The total number of turns is 100. Assuming ideal solenoid, find the flux passing through a circular surface having centre on axis of solenoid of radius 3 cm and is perpendicular to the axis of solenoid (i) inside and (ii) at the end of solenoid.*

Sol.



Number of turns per unit length is given by

$$n = \frac{N}{l} = \frac{100}{4} = 25\ \text{turns/m}$$

(i) Magnetic field of a solenoid at a point inside is

$$B = \mu_0 ni;\ S = \pi r_1^2 \text{ and } \theta = 0°$$

Magnetic flux

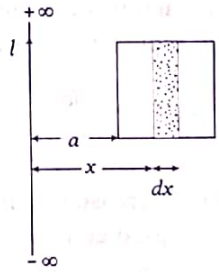
$$\phi_B = BS\cos\theta = \mu_0 ni\,\pi r_1^2\cos 0°$$
$$= 4\pi \times 10^{-7} \times 25 \times 3 \times \pi \times (3\times 10^{-2})^2$$
$$= 0.27 \times 10^{-6}\ \text{Wb} = 0.27\mu\ \text{Wb}$$

(ii) At the end, magnetic field of solenoid is given by

$$= \frac{1}{2}\mu_0 ni$$

$$\therefore \quad \phi = \frac{0.27}{2} = 0.135\,\mu\text{Wb}$$

Example 6.3 *A long straight wire carrying current I and a square conducting wire loop of side l , at a distance 'a' from current wire as shown in the figure. Both the current wire and loop are in the plane of paper. Find the magnetic flux of current wire, passing through the loop.*



Sol. As the field of current wire passing through the loop is same in direction (normally inward) but not uniform in magnitude. So, we will use integration method for finding the flux.

The small flux through a thin rectangular strip of length *l* and width *dx*, is given by

$$d\phi_B = \mathbf{B}_x \cdot d\mathbf{S} = B_{(x)}dS\cos 180°.$$

Magnetic field due to a long straight wire carrying current *I* is given by $B = \frac{\mu_0 I}{2\pi x}$, and area, $dS = l \times dx$.

$$\therefore \quad \phi_B = \int d\phi_B = -\int \frac{\mu_0}{2\pi}\frac{Il}{x}\cdot dx = -\frac{\mu_0}{2\pi} Il[\log_e x]_{x=a}^{x=a+l}$$

$$= -\frac{\mu_0}{2\pi}\cdot Il\log_e\frac{a+I}{a}$$

Faraday's laws of electromagnetic induction

There are two laws of electromagnetic induction given by Faraday's law as below:

Faraday's first law

Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in the circuit.

The actual number of magnetic lines passing through the circuit does not matter of the values of the induced emf. Induced current is determined by the rate at which the magnetic flux changes.

Faraday's second law

The magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit. Mathematically, Faraday's second law can be expressed as,

Induced emf $\propto$ Rate of change of magnetic flux

$$e = \frac{-d\phi_B}{dt}$$

$$\left[\because \text{rate of change of magnetic flux} = \frac{\phi_2 - \phi_1}{t_2 - t_1}\right]$$

The negative sign in above relation indicates that the induced emf in the loop due to changing flux always opposes the change in the magnetic flux.

In the case of a closely wound coil of N turns, the change of flux associated with each turn, is same. Therefore, the expression for the total induced emf is given by

$$\boxed{e = -N\frac{d\phi_B}{dt}}$$

The induced emf can be increased by increasing the number of turns N of a closed coil.

The magnetic flux can be changed in the following ways

(i) The magnitude of **B** can change with time.
Thus, time-varying magnetic field is
$B = B(t)$

(ii) The current producing the magnetic field can change with time. Thus, time-varying current is
$i = i(t)$

(iii) The area enclosed by the loop can change with time. This can be done by pulling a loop inside (or outside) a magnetic field. By doing, so the area enclosed by loop (hatched area) can be changed.

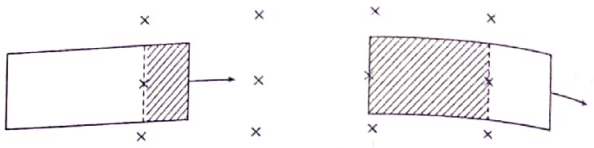


Fig. 6.3

(iv) The angle θ between **B** and the normal to the loop can change with time.
This can be done by rotating a loop in a magnetic field.

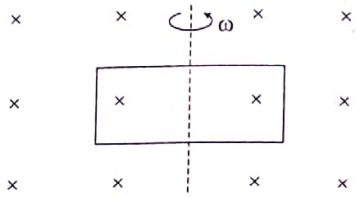


Fig. 6.4

(v) Any combination of the above can occur.

Example 6.4 *A coil with an average diameter of* 0.02 *m is placed perpendicular to a magnetic field of* 6000 *T. If the induced emf is* 11 *V. When the magnetic field is changed to* 1000 *T in* 4 *s. What is the number of turns in the coil?*

Sol. Given, radius of coil, $r = \frac{0.02}{2} = 0.01\,\text{m}$

$B_1 = 6000\ \text{T}, B_2 = 1000\ \text{T}, t = 4\ \text{s}, e = 11\,\text{V}$

$$\text{emf, } e = N\frac{(\phi_2 - \phi_1)}{t} = NA\frac{B_2 - B_1}{t}$$

$$= N\pi r^2\frac{B_2 - B_1}{t}$$

$$11 = N \cdot \frac{22}{7} \times (0.01)^2 \times \frac{(6000 - 1000)}{4}$$

Number of turns,

$$N = \frac{11 \times 7 \times 4}{22 \times (0.01)^2 \times 5000} = 28$$

Induced current and induced charge

Induced current

When the magnetic flux passing through a loop is changed, an induced emf and hence an induced current is produced in the circuit.

If R is the resistance of the circuit, then induced current is given by

$$i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\phi_B}{dt}\right) \quad \ldots(i)$$

If induced current is produced in a coil rotated in a uniform magnetic field, then

$$I = \frac{NBA\,\omega\sin\omega t}{R} = I_0\sin\omega t$$

where, $I_0 = \dfrac{NBA\,\omega}{R}$ = peak value of induced current,

N = number of turns in the coil,

B = magnetic field, ω = angular velocity of rotation,

A = area of cross-section of the coil.

Induced charge

When the current starts flowing in the circuit, flow of charges also takes place. Charge flown in the circuit in time dt will be given by

$$dq = idt = \frac{1}{R}(-d\phi_B) \qquad ...(ii)$$

Thus, for a time interval Δt we can write,

$$e = -\frac{\Delta\phi_B}{\Delta t}, \quad i = \frac{1}{R}\left(\frac{-\Delta\phi_B}{\Delta t}\right)$$

and $$\Delta q = \frac{1}{R}(-\Delta\phi_B) \qquad ...(iii)$$

Direction of induced current

The direction of induced current in a loop may be obtained by using

$$e = -\frac{d\phi}{dt} \text{ and } i = \frac{1}{R}\left(-\frac{d\phi}{dt}\right)$$

So, the steps to decide the direction of induced current are as follows

(i) Define a positive direction for the area vector **S**.

(ii) From the directions of **S** and the magnetic field **B**, determine the sign of ϕ_B and its time rate of change *i.e.*, $\dfrac{d\phi_B}{dt}$.

(iii) If $\dfrac{d\phi_B}{dt}$ is positive, *i.e.*, flux is increasing, sign of induced current is negative and *vice-versa*.
In this way, we will determine the sign of the induced current.

(iv) Finally, determine the direction of induced current using your right hand. Curl the fingers of your right hand around **S**, with your right thumb in the direction of **S**. If the induced current is positive, it is in the same direction as your curled fingers and if it is negative, it is in the opposite direction.

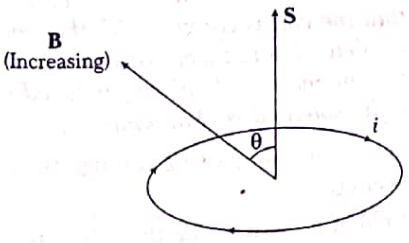


Fig. 6.5 [$\theta < 90°, \phi_B > 0, \dfrac{d\phi_B}{dt} > 0$, *so induced current is negative*]

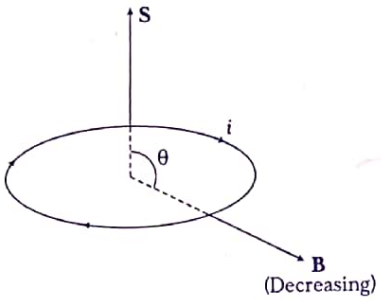


Fig. 6.6

[$\theta > 90°, \phi_B < 0, \dfrac{d\phi_B}{dt} > 0$, so induced current is negative]

Example 6.5 *A coil consists of 200 turns of wire having a total resistance of 2.0 Ω. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.5 T in 0.80 s, then what is the magnitude of induced emf and current in the coil while the field is changing?*

Sol. Given, $N = 200$ turns, $R = 2.0\,\Omega$, change in magnetic field, $\Delta B = 0.5$ T, side of square, $l = 18$ cm $= 18 \times 10^{-2}$ m, $\Delta t = 0.80$ s

From the Faraday's law,

Induced emf, $|e| = -\dfrac{Nd\phi_B}{dt} = (N \times S)\dfrac{\Delta B}{\Delta t}$ $[\because \phi_B = \mathbf{B}\cdot\mathbf{S}]$

Area, $S = l^2 = (18 \times 10^{-2})^2$

$$= \frac{(200)\,(18\times 10^{-2})^2\,(0.5-0)}{0.8} = 4.05 \text{ V}$$

$\therefore$ Further, induced current, $i = \dfrac{|e|}{R} = \dfrac{4.05}{2} \approx 2.0$ A

Example 6.6 *The magnetic flux threading a metal ring varies with time t according to* $\phi_B = 3\,(at^3 - bt^2)$ *T-m*^{2} *with* $a = 2.00\,s^{-3}$ *and* $b = 6.00\,s^{-2}$. *The resistance of the ring is 3.0 Ω. Determine the maximum current induced in the ring during the interval from t = 0 to t = 2.0 s.*

Sol. Given, magnetic flux $\phi_B = 3\,(at^3 - bt^2)$ T-m^2

where, $a = 2\text{s}^{-3}$, $b = 6\text{s}^{-2}$

$\therefore$ Induced emf, $|e| = \left|\dfrac{d\phi_B}{dt}\right| = 9at^2 - 6bt$

$\therefore$ Induced current, $i = \dfrac{|e|}{R} = \dfrac{9at^2 - 6bt}{3} = 3at^2 - 2bt$

For current to be maximum, $\dfrac{di}{dt} = 0$

$\therefore$ $6at - 2b = 0$ or $t = \dfrac{b}{3a}$

i.e., at $t = \frac{b}{3a}$, current is maximum.

Substituting the values in Eq. (b), the maximum current is

$$i_{max} = 3a\left(\frac{b}{3a}\right)^2 - 2b\left(\frac{b}{3a}\right)$$

$$= \frac{b^2}{3a} - \frac{2b^2}{3a} = \frac{-b^2}{3a}$$

Magnitude of this maximum current will be $i_{max} = \frac{b^2}{3a}$.

Substituting the given values of a and b, we have

$$i_{max} = \frac{(6)^2}{3(2)} = 6.0 \text{ A}$$

Example 6.7 *A square loop of side 10 cm and resistance 0.5 Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.10 T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.70 s at a steady state. What is the magnitude of current in this time-interval?*

Sol. Given, $B = 0.10$ T, area of square loop $= 10 \times 10 = 100 \text{ cm}^2$

$= 10^{-2} \text{m}^2$

As the magnetic field is set up across the plane in the north-east direction, then $\theta = 45°$

The initial magnetic flux is given by

$$\phi_B = BS\cos\theta$$

$$\therefore \quad \phi = \frac{0.1 \times 10^{-2}}{\sqrt{2}} \text{ Wb}$$

Final flux, $\phi_{min} = 0$ (given)

The change in flux is brought about in 0.70 s, *i.e.*, $\Delta t = 0.70$ s.

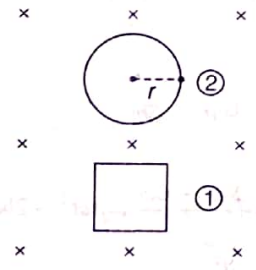
The magnitude of the induced emf is

$$e = \frac{\Delta\phi_B}{\Delta t} = \frac{|\phi - 0|}{\Delta t} = \frac{10^{-3}}{\sqrt{2} \times 0.7} \cong 1 \text{ mV}$$

The magnitude of induced current is $I = \frac{e}{R} = \frac{10^{-3}}{0.5} = 2 \text{ mA}$

Example 6.8 *A wire of length l in the form of a square loop lies in a plane normal to a magnetic field B_0. If this wire is converted into a circular loop in time, then find the average induced emf.*

Sol.



Total length of circle will be $l = 2\pi r \Rightarrow r = \frac{l}{2\pi}$

Side of square, $L = \frac{l}{4}$.

Using the relation, $\phi_B = \mathbf{B} \cdot \mathbf{S} = BS\cos\theta$

Here, $\theta = 0$

$\Rightarrow$ $\phi_B = BS$

Magnetic flux of square loop will be

$$\phi_1 = B_0 L^2 = \frac{B_0 l^2}{16}$$

Magnetic flux of circular loop will be

$$\phi_2 = B_0 \pi r^2 = \frac{B_0 \pi l^2}{4\pi^2} = \frac{B_0 l^2}{4\pi}$$

Average induced emf is given by

$$\bar{e} = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{B_0 l^2}{t_0}\left(\frac{1}{4\pi} - \frac{1}{16}\right) = \frac{B_0 l^2}{16\pi t_0}(4 - \pi)$$

Example 6.9 *Suppose a coil of area 5m^2, resistance 10 Ω and number of turns 200 held perpendicular to a uniform magnetic field of strength 0.4 T. The coil is now turned through 180° in time 1 s. What is*

(i) average induced emf

(ii) average induced current

(iii) total charge that flows through a given cross-section of the coil?

Sol. Given, area of coil, $S = 5\text{m}^2$, resistance, $R = 10\Omega$, number of turns, $N = 200$, magnetic field, $B = 0.4$ T, $\Delta t = 1$ s.

When the plane of coil is perpendicular to the magnetic field *i.e.*, $\theta_i = 0°$ and after it is rotated through 180°, then $\theta_f = 180°$.

$\Rightarrow$ Initial flux, $\phi_i = NBS\cos 0° = NBS$

$= 200 \times 0.4 \times 5 = 400 \text{ Wb}$

and final flux $= NBS\cos 180° = -NBS = -400 \text{ Wb}$

$\Rightarrow$ Change in flux, $|\Delta\phi_B|$

$= NBS - (-NBS) = 2NBS = 800 \text{ Wb}$

(i) Average induced emf $(\varepsilon) = \frac{|\Delta\phi|}{\Delta t} = \frac{2NBA}{\Delta t} = 800 \text{ V}$

(ii) Average current $= \frac{\varepsilon}{R} = \frac{2NBA}{R\Delta t} = 80 \text{ A}$

(iii) Average charge $= \frac{\Delta Q}{\Delta t} = \frac{2NBA}{R\Delta t}$

$\Rightarrow$ Total charge $\Delta Q = \frac{2NBA}{R} = 80 \text{ C}$

Example 6.10 *Through a long solenoid of diameter 4.1 cm, having 100 turns per cm, a current $I = 1A$ is flowing. At its centre, a 60 turns closely packed coil of diameter 3.1 cm is placed such that the coil is coaxial with the long solenoid. The current in the solenoid is reduced to zero at a steady rate in 10 ms. What is the magnitude of emf induced in the coil while the current in the solenoid is changing?*

Sol. Initially, magnetic flux passing through the coil (one turn), $\phi_1 = \mathbf{B} \cdot \mathbf{A} = BA\cos 0°$.

Magnetic field at a point inside the solenoid is given by $B = \mu_0 nI$, where n is number of turns per metre,

$$\therefore \quad \phi = \mu_0 nI \times \frac{\pi d^2}{4}$$

Here, $n = 10000$ turns per metre,

$I = 1\text{A}, d = 3.1$ cm

$$\therefore \quad \phi = \mu_0 nI\frac{\pi d^2}{4} = 4\pi \times 10^{-7} \times 1 \times 10000 \times \pi \times \frac{(3.1 \times 10^{-2})^2}{4}$$

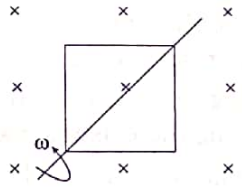
$$= 0.947 \times 10^{-5} \text{ Wb}$$

Finally, the flux becomes zero because the current reduces to zero.

Thus, induced emf, $e = \frac{|\Delta\phi|}{\Delta t} = \frac{0.947 \times 10^{-5}}{10 \times 10^{-3}} = 9.47 \times 10^{-4}\text{V}$

The total emf $= N \times e = 60 \times 9.47 \times 10^{-4} = 568.2 \times 10^{-4}\text{V}$

Example 6.11 *A square loop of edge b having M turns is rotated with a uniform angular velocity ω about one of its diagonals which is kept fixed in a horizontal position. A uniform magnetic field B_0 exists in the vertical direction.*



Find (i) the emf induced in the coil as a function of time t.

(ii) the maximum emf induced.

(iii) the average emf induced in the loop over a long period.

(iv) if resistance of loop is R, amount of charge flown in time $t = 0$ to $t = 2T$.

(v) heat produced in time $t = 0$ to $t = 2T$.

Sol. (i) Initially, plane of loop is $\perp^{ar}$ to the field. Let at any time *t*, normal to loop makes an angle θ with magnetic field $\theta = \omega t$.

The flux passing through loop,

$$\phi_B = MB_0 S\cos\theta = MB_0 b^2 \cos\omega t$$

$$\Rightarrow \quad e = -\frac{d\phi_B}{dt} = MB_0 b^2 \omega \sin\omega t = e_0 \sin\omega t,$$

where, $e_0 = MB_0 b^2 \omega$

(ii) For emf to be maximum, then $\sin\omega t$ should be equal to 1.

$$\Rightarrow \quad e_{max} = e_0$$

(iii) Long period means one time period, *i.e.*, $0 \to T$

We know that average emf is given as

$$\bar{e} = \frac{\int_0^T e dt}{\int_0^T dt} = \frac{1}{T} \cdot e_0 \int_0^T \sin\omega t dt = 0 \qquad \left(\because \int_0^T \sin\omega t dt = 0\right)$$

(iv) Using the relation, induced current,

$$i = \frac{e}{R} = \frac{e_0}{R}\sin\omega t = i_0 \sin\omega t$$

where, $i_0 = \frac{e_0}{R}$.

We know, $q = idt$

So, for time $0 \to 2T$,

$$q = \int_0^{2T} idt = i_0 \int_0^{2T} \sin\omega t dt = 0$$

(v) Heat (*H*) generated in a loop during time $0 \to 2T$, is given as

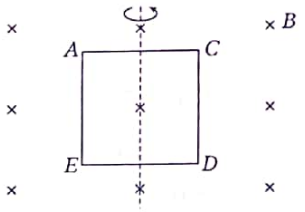
$$H = \int_0^{2T} \frac{e^2}{R} dt = \frac{e_0^2}{0} \int_0^{2T} \sin^2\omega t dt$$

$$= \frac{e_0^2}{R} \cdot 2 \cdot \frac{T}{2} = \frac{e_0^2}{R} \cdot \frac{2\pi}{\omega} = \frac{2\pi e_0^2}{\omega R} \qquad \left(\because T = \frac{2\pi}{\omega}\right)$$

Example 6.12 *A square loop ACDE of area 20 cm^2 and resistance 5 Ω is rotated in a magnetic field* $\mathbf{B} = 2$ *T through* 180°,

(i) in 0.01 s and *(ii) in 0.02 s.*

Find the magnitude of e, i and Δq in both the cases.



Sol. Let us take the area vector **S** perpendicular to plane of loop inwards. So initially, $d\mathbf{S} \uparrow\uparrow \mathbf{B}$ and when it is rotated by 180°, $\mathbf{S} \uparrow\downarrow \mathbf{B}$.

Hence, initial flux passing through the loop,

$$\phi_i = BS\cos 0° = (2)(20 \times 10^{-4})(1)$$

$$= 4 \times 10^{-3} \text{ Wb}$$

Flux passing through the loop when it is rotated by 180°,

$$\phi_f = BS\cos 180°$$

$$= (2)(20 \times 10^{-4})(-1) = -4 \times 10^{-3} \text{ Wb}$$

Therefore, change in flux,

$$\Delta\phi_B = \phi_f - \phi_i = -8 \times 10^{-3} \text{ Wb}$$

(i) Given at $\Delta t = 0.01$ s, $R = 5\,\Omega$

Using the relation,

$$|e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8 \times 10^{-3}}{0.01} = 0.8 \text{ V}$$

or induced current $i = \frac{|e|}{R} = \frac{0.8}{5} = 0.16$ A

and charge $\Delta q = i\Delta t = 0.16 \times 0.01 = 1.6 \times 10^{-3}$ C

(ii) Similarly, given at $\Delta t = 0.02$ s, $R = 5\,\Omega$

$$\therefore \quad |e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8 \times 10^{-3}}{0.02} = 0.4 \text{ V}$$

$$\therefore \quad i = \frac{|e|}{R} = \frac{0.4}{5} = 0.08 \text{ A}$$

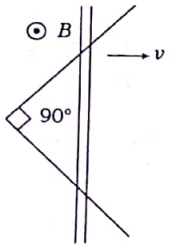
and charge $\Delta q = i\Delta t = (0.08)(0.02) = 1.6 \times 10^{-3}$ C

Example 6.13 *The two conducting rails are placed perpendicular to each other, such that their ends are joined as shown in figure. A conducting bar is now placed over the rails and start moving with constant velocity v starting from the vertex at time t = 0.*

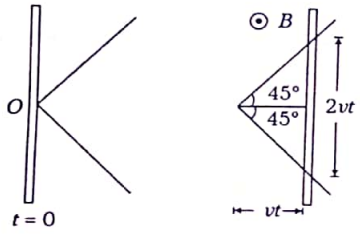
(i) The flux through the triangle (isosceles) by the rails and bar at $t = t_0$.

(ii) The emf around the triangle at that time.

(iii) In what manner does the emf around the triangle vary with time.



Sol.



(i) At time $= t$, area of loop, $S = \frac{1}{2} \cdot vt \cdot 2vt = v^2t^2$

($\because$ area of isosceles triangle $= \frac{1}{2} \times$ base $\times$ height)

Using the relation,

Magnetic flux $\phi_B = BS = Bv^2t^2$ and at, $t = t_0 \Rightarrow \phi = Bv^2t_0^2$

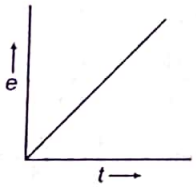
(ii) The emf developed around the triangle at to,

further, $e = \left|\frac{d\phi_B}{dt}\right| = Bv^2 \cdot 2t = 2Bv^2t$

At $t = t_0$, $e = 2Bv^2t_0$

(iii) According to the relation obtained for the emf

i.e., $e = 2Bv^2t \Rightarrow e \propto t$



Hence, emf varies linearly with time.

Lenz's law and conservation of energy

The negative sign in Faraday's equation of electromagnetic induction describes the direction in which induced emf drives the current. This direction is easily determined with the help of Lenz's law, an associated principle with Faraday's law.

Lenz's law give the same result as the sign rule we introduced in Faraday's law but is often easier to use. Lanz's law states that, **the direction of any magnetic induction effect is such as to oppose the cause of effect.**

This law is based upon law of conservation of energy. As the induced emf opposes the change in flux, work has to be done against the opposition offered by induced emf/current in changing the flux. The work done appears as electrical energy in the circuit.

Direction of induced current with the help of Lenz's law

Direction of induced current can be determined by checking whether the flux through a conducting loop or circuit is increasing or decreasing.

- If flux is decreasing, the magnetic field due to induced current will be along the existing magnetic field.
- If flux is increasing, the magnetic field due to induced current will be opposite to existing magnetic field.

Important points based on Lenz's law

The following are some important points that will explain the direction of induced current according to Lenz's law.

(i) Consider a conducting loop is being pulled inside a magnetic field, hence area of loop inside the magnetic field increases.

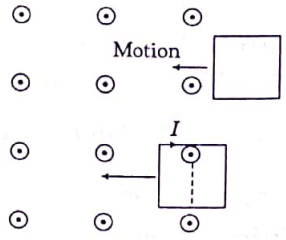


Fig. 6.7

From the above figure, it is clear that the magnetic flux through loop is increasing and hence magnetic field due to induced current will be opposite to the existing magnetic field *i.e.*, inside the plane of paper, thus direction of induced current will be clockwise.

(ii) Consider a loop is being pulled out of magnetic field, area of loop inside field decreases and hence flux ($\phi = BS$) passing through loop also decreases.

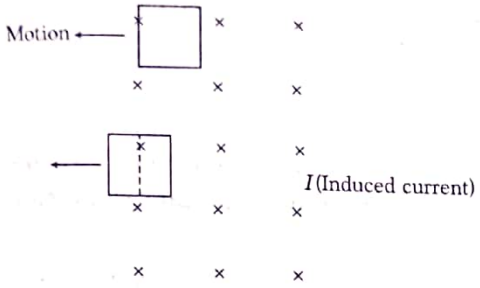


Fig. 6.8

So, magnetic field due to induced current will be along the existing magnetic field, *i.e.*, inside the plane of paper, thus direction of induced current will be clockwise.

(iii) Consider a loop is placed near a current carrying loop wire. The magnetic field due to long wire at distance x from it is given by

$$B = \frac{\mu_0 i}{2\pi x}$$

$$\Rightarrow \quad B \propto i$$

$$\Rightarrow \quad \phi \propto B \propto i$$

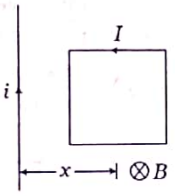


Fig. 6.9

If the current in long wire increases, then ϕ also increases. The direction of magnetic field due to long wire will be ⊗. In this case, ϕ is increasing, magnetic field due to induced current will be opposite to existing magnetic field, *i.e.*⊙, hence direction of induced current is anti-clockwise.

(iv) If a conducting loop is brought away from a current carrying straight wire.

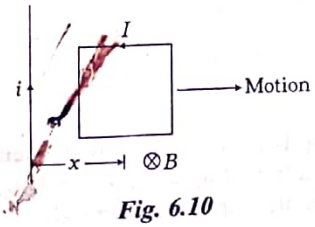


Fig. 6.10

As the loop is moving away so, flux through the loop will decrease, magnetic field due to induced current will be along the existing magnetic field, *i.e.*, ⊗, hence direction of induced current is clockwise.

(v) Consider two conducting loops are placed in a magnetic field.

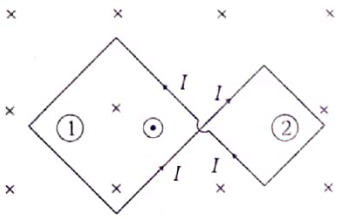


Fig. 6.11

In this case, if magnetic field is increasing, then flux through each loop will increase, hence magnetic field due to induced current will be opposite to the existing magnetic field, *i.e.* ⊙, hence induced current will be anti-clockwise but single current will flow in the wire. The induced emf in loop 1 will be greater due to its bigger area, so single current will flow according to current in loop 1. Induced current in 1 will be anti-clockwise and in loop 2 will be clockwise.

(vi) (a) When north pole moves towards ring, then flux will increase, induced current will oppose this, so north pole will be formed in loop as seen by observer.

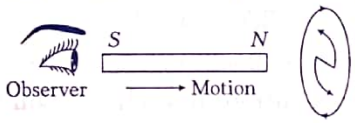


Fig. 6.12

Induced current will be anti-clockwise.

(b) When north pole moves away from ring, then flux will decrease, induced current will oppose this, so south pole will be formed in loop as seen by observer.

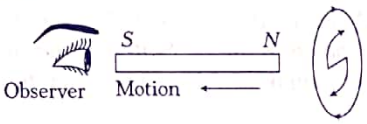


Fig. 6.13

Induced current will be clockwise.

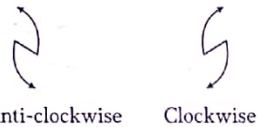


Fig. 6.14

(c) Similar observations (as in case (a) can be observed when south pole moves towards ring. So, induced current will be clockwise here.

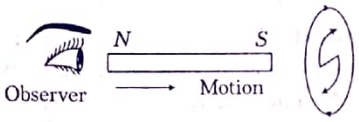


Fig. 6.15

(d) Similar observations (as in case (b) can be observed when south pole moves away from the ring. So, induced current in this case will be anti-clockwise.

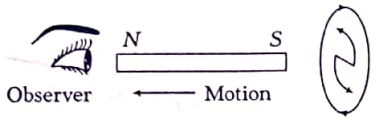


Fig. 6.16

(vii) Two co-axial circular loops, identical to each other, carry equal amount of currents and circulating in the same direction.

(a) When loops approaches each other, then the flux linked with each coil will increase.

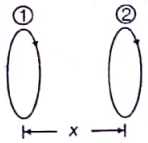


Fig. 6.17

Magnetic field due to 1 ① at ② will be given by

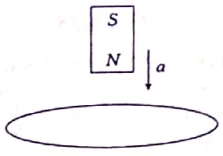
$$B = \frac{\mu_0 NiR^2}{2(R^2 + x^2)^{3/2}}$$

So, x decreases, B will increase ϕ also increases.

The induced current in each coil will try to reduce flux and will be opposite to original current. So, current in each coil will decrease.

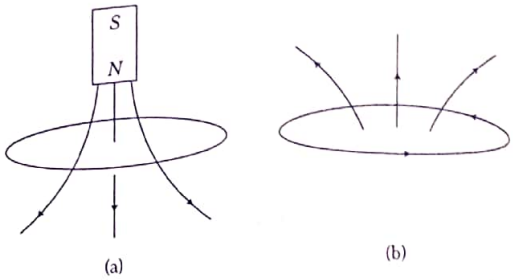
(b) Now, loops move away from each other, then the flux linked with coil will decrease and so the induced current will try to increase the flux and hence current in each coil will increase.

Example 6.14 *A bar magnet is freely falling along the axis of a circular loop as shown in figure. State whether its acceleration 'a' is equal to, greater than or less than the acceleration due to gravity g.*



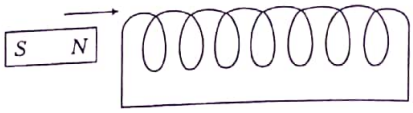
Sol. Acceleration of the bar magnet will be less than acceleration due to gravity ($a < g$) because according to Lenz's law, whatever may be the direction of induced current, it will oppose the cause. Here, the cause is, the free fall of magnet and so the induced current will oppose it and the acceleration of magnet will be less than the acceleration due to gravity g.

Alternate method This can be understood in a different manner. When the magnet falls downwards with its north pole downwards. The magnetic field lines passing through the coil in the downward direction increase. Since, the induced current opposes this, the upper side of the coil will become north pole, so that field lines of coil's magnetic field are upwards. Now, like poles repel each other. Hence, $a < g$.

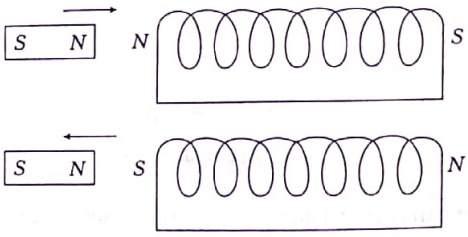


If this circular loop had a slot, an emf in that case will be induced but no current will flow *i.e.*, $a = g$.

Example 6.15 *A bar magnet is brought near a solenoid as shown in figure. Will the solenoid attract or repel the magnet?*

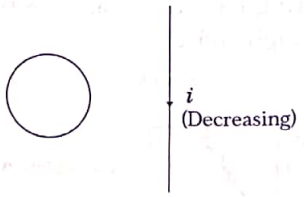


Sol. When the magnet is brought near the solenoid, then according to Lenz's law, both repel each other. On the other hand, if the magnet is moved away from the solenoid, then it attracts the magnet. When the magnet is brought near the solenoid, then the nearer side becomes the same pole and when it is moved away it becomes the opposite pole as shown in figure.



It can also be understood by increasing or decreasing field lines as in Example 6.14.

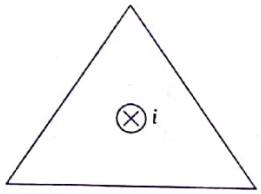
Example 6.16 *A circular loop is placed near a current carrying conductor as shown in figure. Find the direction of induced current, if the current, in the wire is decreasing.*



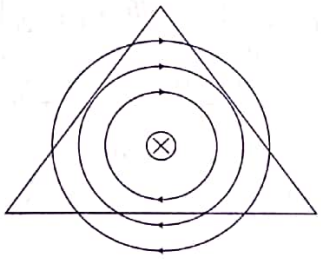
Sol. In this case loop is placed to the right of current carrying wire (not to the left as it appears, because if you move in the direction of current, loop lies to the right).

Now, the current is decreasing, therefore induced current in the loop is clockwise (S) as shown in figure.

Example 6.17 *A current carrying straight wire passes inside a triangular coil as shown in figure. The current in the wire is perpendicular to paper inwards. Find the direction of the induced current in the loop, if current in the wire is increased.*

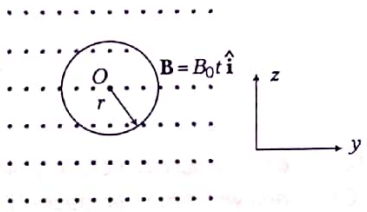


Sol. Magnetic field lines around the current carrying wire are as shown in figure below. Since, the lines are tangential to the loop ($\theta = 90°$), the flux passing through the loop is zero, whether the current is increased or decreased. Hence, change in flux is zero. Therefore, induced current in the loop will be zero.



Example 6.18 *A uniform magnetic field* $\mathbf{B} = B_0 t\hat{\mathbf{i}}$ *in a region exists. A circular conducting loop of radius r and resistance R is placed with its plane in yz-plane. Determine the current through the loop and sense of the current.*

Sol. The flux linked with the loop is



We know that magnetic flux is given as

$$\phi_B = \mathbf{B} \cdot \mathbf{S} = (B_0 t)(\pi r^2)\cos 0°$$

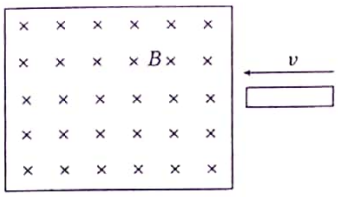
$$\Rightarrow \quad \phi_B = B_0 \pi r^2 t \quad \text{and} \quad \frac{d\phi}{dt} = B_0 \pi r^2$$

Further, induced emf, $\varepsilon = \dfrac{-d\phi}{dt} = -B_0 \pi r^2$

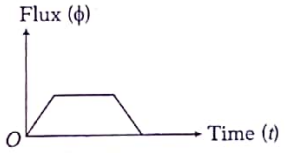
Therefore, current, $i = \dfrac{|\varepsilon|}{R} = \dfrac{B_0 \pi r^2}{R}$

By Lenz's law, i should be clockwise, so that it can oppose the increase in magnetic field.

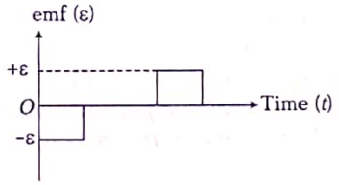
Example 6.19 *Through a uniform B field, A small rectangular loop is moving towards left with constant velocity as shown in the figure. Counting of time t begins at the moment, the loop starts entering the field. Plot the variation of flux through the loop with respect to time. Also, plot the variation of induced emf w.r.t. time t.*



Sol. As the loop is moving with constant velocity, the flux will be increasing linearly, as long as the loop will be entering the B field. Once the loop has completely entered the B field the flux will become constant. Now, when the loop will start coming out the flux will linearly decrease to zero value.

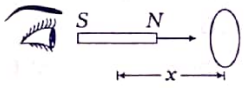


Since, emf induced, $\varepsilon = -\dfrac{d\phi}{dt}$, it means emf is negative of the slope of the ϕ-t graph. So, emf is negative constant in the beginning, then it becomes zero and then it becomes positive constant and again it becomes zero.



Example 6.20 *Through a conducting coil along its axis, a short bar magnet is rapidly pulled with uniform velocity with, with its north pole entering the coil first. Plot the variation of (i) flux, (ii) induced current and (iii) power dissipated in coil with time.*

Sol. (i)

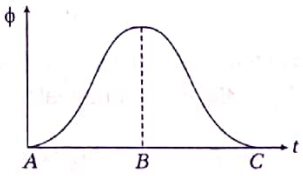


Magnetic field due to magnet is given as

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3}$$

$$\Rightarrow \quad B \propto \frac{1}{x^3} \Rightarrow \phi \propto \frac{1}{x^3}$$

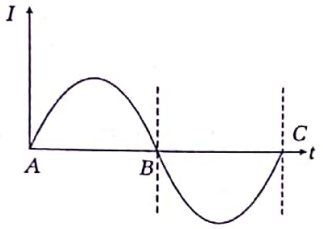
When a bar magnet is pulled rapidly through a conducting coil, then flux first increases, reaching to a maximum value and then decreases.



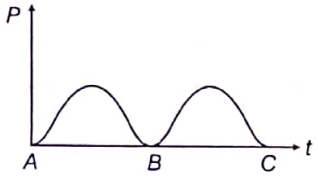
(ii) *Induced current versus time t*

When flux increases, then north pole is formed, current is anti-clockwise.

When flux decreases, then south pole is formed, current is clockwise. (Taking anti-clockwise current as positive.)

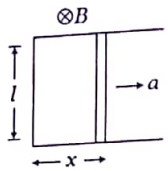


(iii) As power ∝ induced current. The corresponding power *versus t* can be drawn as



Example 6.21 *In a uniform magnetic field, a π shaped metal frame is located perpendicular to the plane of the conductor and varying with time at the rate* $(dB/dt) = 0.20\,T/s$. *A conducting connector starts moving with an acceleration* $a = 30\ cm/s^2$ *along the parallel bars of the frame. The length of the connector is equal to* $l = 44\ cm$. *Find the emf induced in the loop* $t = 1\ s$ *after the beginning of the motion, if at the moment* $t = 0$, *the loop area and the magnetic induction are equal to zero.*

Sol.



We know, according to kinematic equation, distance travelled $x = ut + \frac{1}{2}at^2$. Here, initial velocity, $u = 0$, acceleration, $a = b = 30\ cm/s^2$

So, in time t, distance travelled by conductor, $x = \frac{1}{2}bt^2$

Flux passing through loop, $\phi = BS = Blx = \frac{Blbt^2}{2}$

Here, B and t both are variable.

So, induced emf, $e = \left|\frac{d\phi}{dt}\right| = \frac{lb}{2}\left[\frac{dB}{dt}t^2 + B \cdot 2t\right]$

$$= \frac{0.4 \times 0.3}{2}(0.2 \times 2^2 + 2B)$$

$$= 0.06\,(0.8 + 2B)$$

Given, $\frac{dB}{dt} = 0.2\ T/s \Rightarrow dB = 0.2dt$

On integrating both sides, we get

$$\int_0^B dB = 0.2\int_0^t dt \Rightarrow B = 0.2\,t = 0.2 \times 1 = 0.2$$

So, induced emf, $e = 0.06(0.8 + 0.2) = 0.06\ V = 6 \times 10^{-2}\ V$

Check point 6.1

1. The magnetic flux linked with a vector area **A** in a uniform magnetic field **B** is

(a) $\mathbf{B} \times \mathbf{A}$ (b) AB (c) $\mathbf{B} \cdot \mathbf{A}$ (d) $\frac{B}{A}$

2. The unit of magnetic flux is

(a) Wbm^{-2} (b) Wb (c) H (d) Am^{-1}

3. The dimensions of magnetic flux are

(a) $[MLT^{-2}A^{-2}]$ (b) $[ML^2T^{-2}A^{-2}]$

(c) $[ML^2T^{-1}A^{-2}]$ (d) $[ML^2T^{-2}A^{-1}]$

4. The magnetic flux linked with a coil, in webers is given by the equation $\phi = 3t^2 + 4t + 9$. Then, the magnitude of induced emf at $t = 2$ s will be

(a) 2 V (b) 4 V (c) 8 V (d) 16 V

5. A coil having an area A_0 is placed in a magnetic field which changes from B_0 to $4B_0$ in time interval t. The emf induced in the coil will be

(a) $3A_0B_0/t$ (b) $4A_0B_0/t$ (c) $3B_0/A_0t$ (d) $4B_0/A_0t$

6. The magnetic flux ϕ (in weber) in a closed circuit of resistance 10 Ω varies with time t (in second) according to equation $\phi = 6t^2 - 5t + 1$. The magnitude of induced current at $t = 0.25$ s is

(a) 1.2 A (b) 0.8 A (c) 0.6 A (d) 0.2 A

7. When the current through a solenoid increases at a constant rate, then the induced current

(a) is a constant and is in the direction of the inducing current

(b) is a constant and is opposite to the direction of the inducing current

(c) increase with time and is in the direction of inducing current

(d) increase with time and is opposite to the direction of inducing current

8. The magnetic flux across a loop of resistance 10 Ω is given by $\phi = 5t^2 - 4t + 1$ Wb. How much current is induced in the loop after 0.2 s?

(a) 0.4 A (b) 0.2 A

(c) 0.04 A (d) 0.02 A

9. A circular ring of diameter 20 cm has a resistance 0.01 Ω. How much charge will flow through the ring, if it is rotated from a position perpendicular to a uniform magnetic field of $B = 2$ T to a position parallel to field?
(a) 4 C (b) 6.28 C (c) 3.14 C (d) 2.5 C

10. In a circuit with a coil of resistance 2 Ω, the magnetic flux changes from 2.0 Wb to 10.0 Wb in 0.2 s. The charge that flows in the coil during this time is
(a) 5.0 C (b) 4.0 C (c) 1.0 C (d) 0.8 C

11. The direction of induced emf during electromagnetic induction is given by
(a) Faraday's law (b) Lenz's law
(c) Maxwell's (d) Ampere's law

12. Two different loops are concentric and lie in the same plane. The current in the outer loop is clockwise and increasing with time. The induced current in the inner loop, is
(a) clockwise
(b) zero
(c) counter-clockwise
(d) in a direction that depends on the ratio of the loop radii

13. Lenz's law is due to conservation of
(a) charge
(b) momentum
(c) energy
(d) current

14. The north pole of a long horizontal bar magnet is being brought closer to a vertical conducting plane along the perpendicular direction. The direction of the induced current in the conducting plane will be
(a) horizontal
(b) vertical
(c) clockwise
(d) anti-clockwise

15. There is a uniform magnetic field directed perpendicular and into the plane of the paper. An irregular shaped conducting loop is slowly changing into a circular loop in the plane of the paper. Then,
(a) current is induced in the loop in the anti-clockwise direction
(b) current is induced in the loop in the clockwise direction
(c) AC is induced in the loop
(d) No current is induced in the loop

2 Motional electromotive force

So far we have considered the cases in which an emf is induced in a stationary circuit placed in a time-varying magnetic field. In this section, we will study about **motional emf,** *i.e.*, the emf induced in a conductor when it is moving through a constant magnetic field. Now, let us consider, a straight conductor of length l as shown in Fig. 6.17 which is moving through a uniform magnetic field directed into the page. For simplicity, we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent.

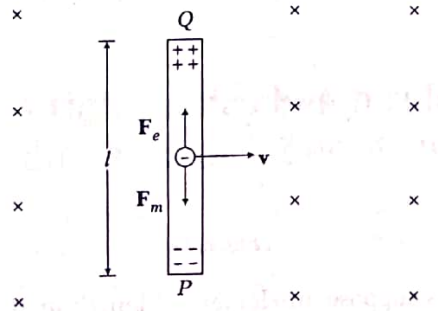


Fig. 6.17

Due to motion of conductor in magnetic field a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.

This potential difference is called **motional electromagnetic force,** which is denoted by e, and is given by

$$\boxed{e = Bvl} \quad \text{...(i)}$$

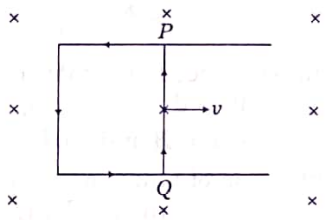


Fig. 6.18

If R is the resistance of the circuit, then current will be written as

$$\Rightarrow \quad i = \frac{e}{R} = \frac{Bvl}{R} \quad \text{...(ii)}$$

General form of motional emf

To find motional emf for a conductor of any shape, moving in a uniform or non-uniform magnetic field, we will consider any small element $d\mathbf{l}$ of a conductor, then the contribution de to the emf is the magnitude of $d\mathbf{l}$ multiplied by the component of $(\mathbf{v} \times \mathbf{B})$ which is parallel to $d\mathbf{l}$, *i.e.*,

$$de = (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\Rightarrow \quad e = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} \quad \text{[motional emf, closed conducting loop]}$$

Important points related to motional emf

Some important points related to motional emf are given below.

(i) For a semicircular conducting loop of radius R with the centre at O and moving with velocity v, then the emf is given by

$$e = Bv(2R)$$

$$\Rightarrow \quad V_P - V_Q = 2BvR$$

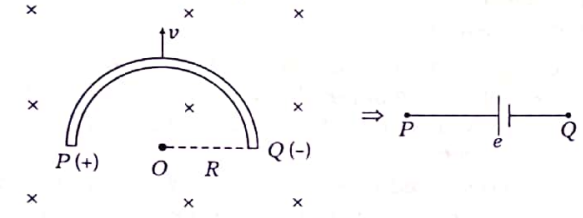


Fig. 6.19

(ii) If a conducting rod AB moves with a velocity v in xy-plane, then we can resolve the velocity vector in two components as shown in Fig. 6.20.

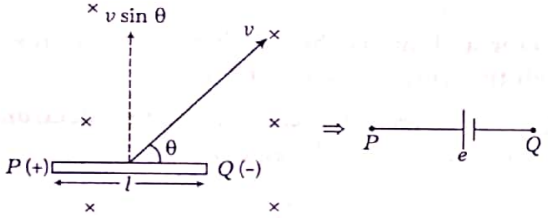


Fig. 6.20

No emf will be induced, due to $v \cos\theta$. So, the net effect of emf will be due to $v \sin\theta$,
which is given by $\boldsymbol{e = B(v \sin\theta)l}$.

(iii) For a circular loop of radius R moving with velocity v, we can replace its two ends lying on opposite sides of diameter with a battery of emf e. Then, the emf is given by

$$\boldsymbol{e = Bv(2R) = 2BvR}$$

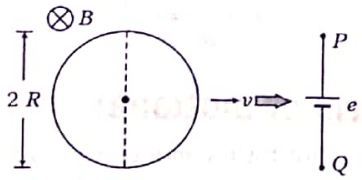


Fig. 6.21

(iv) For a irregular shape, a conducting body of length l, we can assume it, as a straight conductor and replace it with a battery of emf e as shown in Fig. 6.22.

Then, $\boldsymbol{e = B(v \cos\theta)l}$

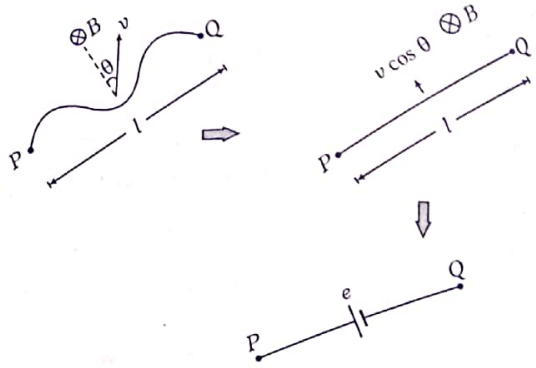


Fig. 6.22

(v) When magnetic field varies at every point of conductor which is moving with speed v, then the induced emf will be given by

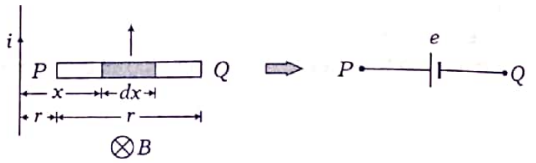


Fig. 6.23

$$de = Bv\,dx,$$

where, B is the magnetic field due to a long wire at distance x and is equal to $\frac{\mu_0 i}{2\pi x}\otimes$.

$$\Rightarrow \quad e = \frac{\mu_0 iv}{2\pi}\int_r^{r+l}\frac{dx}{x} = \frac{\mu_0 iv}{2\pi}\ln\left(1+\frac{l}{r}\right)$$

$$\Rightarrow \quad V_P - V_Q = e$$

Motional emf induced in a rotating bar

(i) A conducting rod of length l rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field B is directed perpendicular to the plane of rotation as shown in Fig. 6.24.

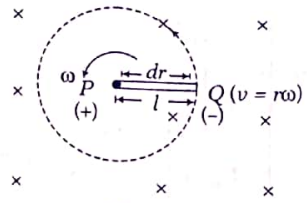


Fig. 6.24

Let us suppose an element of length dr at a distance r from P. The induced emf in this segment is

$$de = Bvr\,dr = B(r\omega)dr$$

Because every segment of the rod is moving perpendicular to B, an emf de of same form is generated across each.

So, summing the emfs induced across the ends of the rod is

$$e = \int_0^l de = \int_0^l Br\,\omega\,dr$$

$$\Rightarrow \quad e = \frac{1}{2}B\omega l^2$$

$$= Bl^2\pi v = \frac{Bl^2\pi}{T} \qquad [\because 2\pi v = \omega]$$

where, v = frequency (revolution per second),

T = time period

(ii) If a conducting disc of radius r rotates with constant angular velocity ω about its axis in a uniform magnetic field parallel to its axis of rotation as shown in Fig. 6.25.

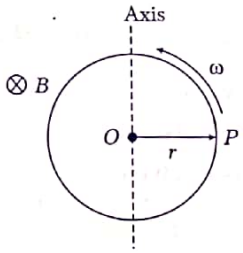


Fig. 6.25

Take a rod between centre and point P.

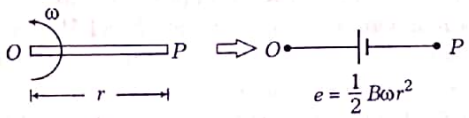


Fig. 6.26

As, we have already deduced the emf for a rod rotating in a constant magnetic field.

So, the induced emf is given by, $e = \frac{1}{2}B\omega r^2$.

That means, disc is equivalent to rod.

Fleming's right hand thumb rule to find the direction of induced current

The direction of motional emf or current can be given by Fleming's right hand thumb rule. If we stretch the thumb, the forefinger and the central finger of right hand in such a way that all are mutually perpendicular to each other and if thumb represents the direction of motion of the conductor, the forefinger represents the direction magnetic field, then central finger will represent the direction of induced current as shown in Fig. 6.27.

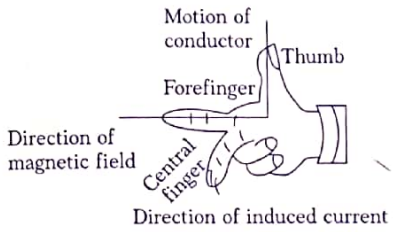
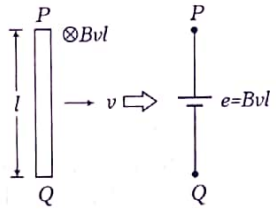


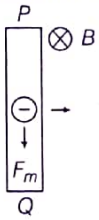
Fig. 6.27

Example 6.22 *A conducting rod of unit length moves with a velocity of 5m/s in a direction perpendicular to its length and perpendicular to a uniform magnetic field of magnitude 0.4T. Find the emf induced between the ends of the stick.*

Sol.



The negative terminal is that end towards which electrons are moving due to magnetic force and, the magnetic force on electron is acting along Q, hence Q is negative terminal of battery.

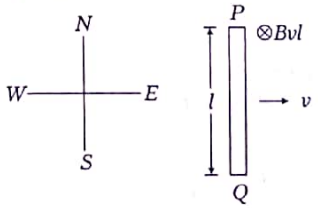


As, we know that induced emf for a moving rod is given by

$$\Rightarrow \qquad e = Bvl = 0.4 \times 5 \times 1 = 2\ \text{V}$$

Example 6.23 *The horizontal component of the earth's magnetic field at a place is 4.0×10^{-4} T and the dip is 45°. A metal rod of length 20 cm is placed in the north-south direction and is moved at a constant speed of 5 cm/s towards east. Calculate the emf induced in the rod.*

Sol.



According to the question, the velocity v, length l and B_v (vertical component of earth magnetic field) are mutually perpendicular using the relation,

$$e = B_v vl \qquad \left[\because \tan\phi = \frac{B_v}{B_h}\right]$$

$$= B_h \tan\phi vl$$

$$= 4 \times 10^{-4} \tan 45° \times 0.05 \times 0.20 = 4 \times 10^{-6}\ \text{V}$$

There will be no induced emf due to B_h (horizontal component of earth magnetic field) because it is parallel to length of rod.

Example 6.24 *A ring of radius 2m translates in its plane with a constant velocity 5m/s. A uniform magnetic field 0.1 T exists in the space in a direction perpendicular to the plane of the ring. Consider different pairs of diametrically opposite points on the ring.*

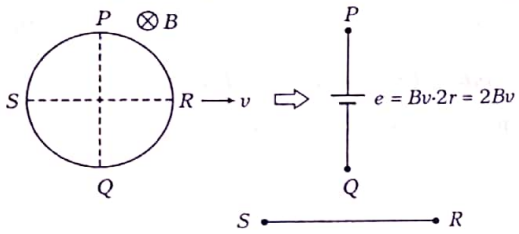
(i) Between which pair of points is the emf maximum? What is the value of this maximum emf?

(ii) Between which pair of points is the emf maximum? What is the value of this minimum emf?

Sol. Given, radius, $r = 2\,m$, velocity, $v = 5\,ms^{-1}$

Constant magnetic field, $B = 0.1\,T$

According to the question,



From figure, we can see that emf is maximum at PQ and minimum at RS as PQ is perpendicular to field and RS along the field.

Using the relation for induced emf for a rod,

$$V_A - V_B = 2Bvr$$

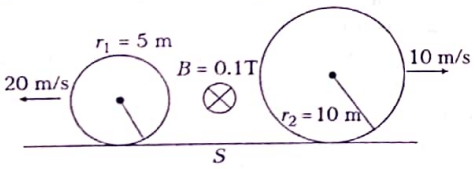
$$e_{max} = 2Bvr$$

$$= 2 \times 0.1 \times 5 \times 2 = 2\,V$$

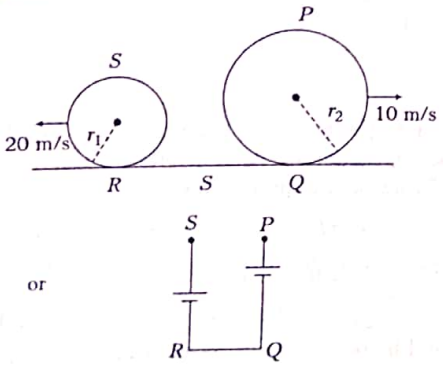
$$V_C - V_D = 0$$

$$\Rightarrow \quad e_{min} = 0$$

Example 6.25 *Two rings of radii 5m and 10m move in opposite directions with velocity 20 m/s and 10 m/s respectively, on a conducting surface S. There is a uniform magnetic field of magnitude 0.1 T perpendicular to the plane of the rings. Find the potential difference between the highest points of the two rings.*



Sol. The above situation can be drawn as



As we know, induced emf in a rod is given by $e = Bvl$.

So, for rod PQ, $e = Bvl = 0.1 \times 10 \times 10 = 10\,V$

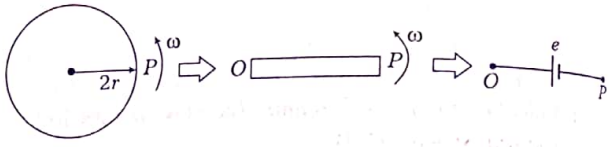
For rod RS, $e = Bvl = 0.1 \times 20 \times 5 = 10\,V$

So, the net emf produced in above figure is given by

$$V_P - 10V - 10V = V_S \;\Rightarrow\; V_P - V_S = 20\,V$$

Example 6.26 *A vertical disc of diameter 10 cm makes 20 revolutions per second about a horizontal axis passing through its centre. A uniform magnetic field 10^{-1} T acts perpendicular to the plane of the disc. Calculate the potential difference between its centre and rim in volts.*

Sol. As we know, disc is equivalent to a rod of length r. Divide disc into rods, each rod can be replaced by a battery and these batteries are would be parallel in arrangement.



According to the question, radius of disc,

$$r = \frac{10}{2} = 5\,cm = 0.05\,m$$

Angular velocity, $\omega = 20 \times 2\pi = 40\,\pi\,rad/s$

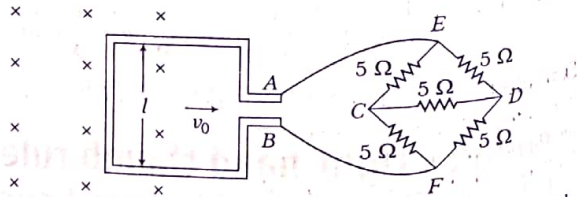
Magnetic field, $B = 10^{-1}\,T$

The emf of a disc is given as

$$e = \frac{1}{2}B\omega r^2 = \frac{1}{2}(10^{-1})(40\pi)(0.05)^2$$

$$\Rightarrow \quad V_O - V_A = 1.57 \times 10^{-2}\,V$$

Example 6.27 *A square metal wire loop of side 20 cm and resistance 2Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 1\ Wb/m^2$ as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop. The loop is connected to a network of resistance each of value 5Ω. The resistances of the lead wires BF and AE are negligible. What should be the speed of the loop, so as to have a steady current of 2 mA in the loop? Give the direction of current in the loop.*



Sol. From the figure, we see that, network $CEOF$ is balanced Wheatstone bridge, so no current will flow in branch CD.

So, the equivalent resistance of $CEOF$ network is

$$R_{eq} = \frac{10 \times 10}{10 + 10} = 5\,\Omega$$

Resistance of loop $= 2\,\Omega$

$$R_{total} = 2 + R_{eq} = 2 + 5 = 7\,\Omega$$

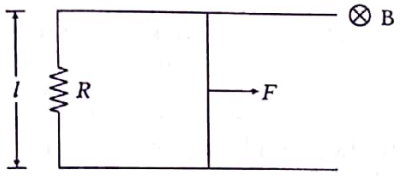
We know that induced emf, $e = Bv_0 l$

and induced current, $I = \frac{e}{R_{total}} = \frac{Bv_0 l}{7} \Rightarrow 2 \times 10^{-3} = \frac{1 \times v_0 \times 0.2}{7}$

$\Rightarrow \quad v_0 = 7 \times 10^{-2}$ m/s = 7 cm/s

As, flux is decreasing, so induced current I will be clockwise.

Example 6.28 *Figure shows the top view of a rod that can slide without friction. The resistor is 6.0 Ω and a 2.5 T magnetic field is directed perpendicularly downward into the paper. Let l =1.20 m.*



(i) *Calculate the force F required to move the rod to the right at a constant speed of 2.0 ms^{-1}.*

(ii) *At what rate is energy delivered to the resistor?*

(iii) *Show that this rate is equal to the rate of work done by the applied force.*

Sol. Given, resistance, $R = 6.0\Omega$

Magnetic field, $B = 2.57$ T

Length of the rod, $l = 1.20$ m

The motional emf in the rod is given as, $e = Bvl$

or $\quad e = (2.5)\,(2.0)\,(1.2)$ V = 6.0 V

The current in the circuit, $i = \frac{e}{R} = \frac{6.0}{6.0} = 1.0$ A

(i) The magnitude of force F required will be equal to the magnetic force acting on the rod, which opposes the motion.

$\therefore \quad F = F_m = lvB = ilB$

or $\quad F = (1.0)\,(1.2)\,(2.5)$ N = 3 N

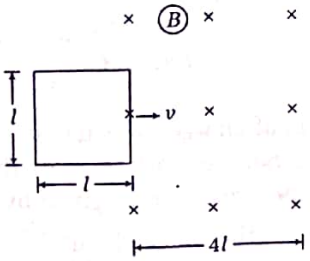
(ii) Rate by which energy is delivered to the resistor is

$$P_1 = i^2R = (1)^2\,(6.0) = 6 \text{ W}$$

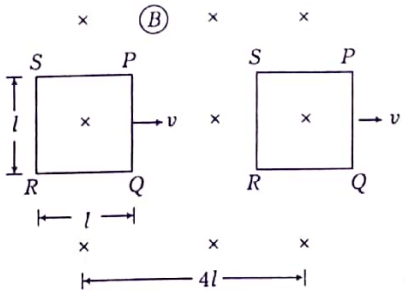
(iii) The rate by which work is done by the applied force is,

$$P_2 = Fv = (3)\,(2.0) = 6 \text{ W} \Rightarrow P_1 = P_2$$

Example 6.29 *A square loop of side l being moved towards right at a constant speed v as shown in figure. The front edge enters the magnetic field B at t = 0. The width of field is 4l. Sketch induced emf versus time graph.*



Sol.



From, $t = 0$ to $t = \frac{l}{v}$

Induced emf,

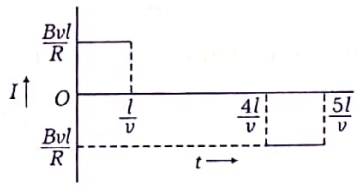
$e = Bvl$ and $I = \frac{e}{R} = \frac{Bvl}{R}$, which is anti-clockwise in direction.

Again from $t = \frac{l}{v}$ to $t = \frac{4l}{v}$, emf $e = 0$

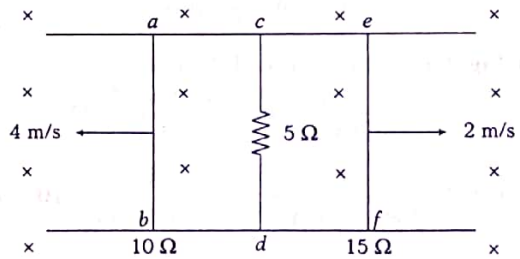
$t = \frac{4l}{v}$ to $t = \frac{5l}{v}$ emf $e = Bvl$

So, induced current $I = \frac{e}{R} = \frac{Bvl}{R}$, which is clockwise in direction now.

So, the plot of induced emf *versus* time is given as



Example 6.30 *Two parallel rails with negligible resistance are 10 cm apart. They are connected by a 5Ω resistor. The circuit also contains two metal rods having resistances of 10 Ω and 15 Ω along the rails. The rods are pulled away from the resistor at constant speeds 4.00 m/s and 2 m/s, respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5 Ω resistor.*



Sol. Here, two conductors are moving in uniform magnetic field. So, we will use the motional approach. The rod *ab* will act as a source of emf,

$$e_1 = Bvl = (0.01)\,(4)\,(0.1) = 4 \times 10^{-3} \text{ V}$$

and internal resistance, $r_1 = 10\Omega$

Similarly, rod *ef* will also act as a source of emf,

$$e_2 = (0.01)\,(2)\,(0.1) = 2 \times 10^{-3} \text{ V}$$

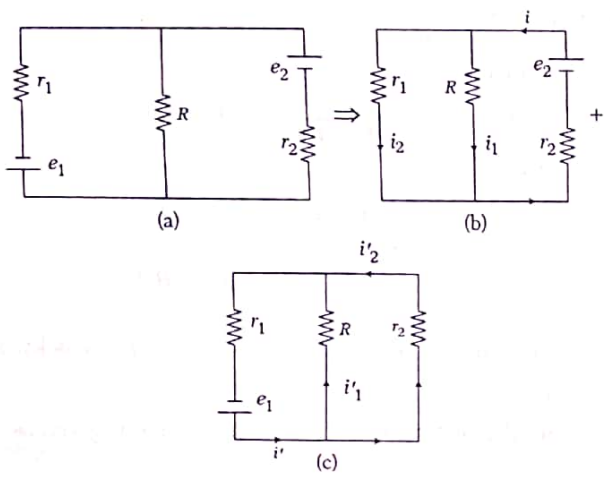
and internal resistance, $r_2 = 15\Omega$

From right hand rule we can see that

$$V_b > V_a \quad \text{and} \quad V_e > V_f$$

Now, either by applying Kirchhoff 's laws or applying principle of superposition (discussed in the chapter of current electricity) we can find current through 5 Ω resistor. We will here use the superposition principle. You solve it by using Kirchhoff 's laws.

Now, in the figures, $R = 5\,\Omega$, $r_1 = 10\,\Omega$, $r_2 = 15\,\Omega$, $e_1 = 4 \times 10^{-3}$ V and $e_2 = 2 \times 10^{-3}$ V.



Refer Fig. (b) Net resistance of the circuit

$$= r_2 + \frac{Rr_1}{R + r_1} = 15 + \frac{10 \times 5}{10 + 5} = \frac{55}{3}\,\Omega$$

$$\therefore \text{Current,} \quad i = \frac{e_2}{\text{Net resistance}}$$

$$= \frac{2 \times 10^{-3}}{55/3} = \frac{6}{55} \times 10^{-3}\ \text{A}$$

∴ Current through R,

$$i_1 = \left(\frac{r_1}{R + r_1}\right) i = \left(\frac{10}{10 + 5}\right)\left(\frac{6}{55} \times 10^{-3}\right)\text{A}$$

$$= \frac{4}{55} \times 10^{-3}\ \text{A} = \frac{4}{55}\ \text{mA}$$

Refer Fig. (c) Net resistance of the circuit

$$= r_1 + \frac{Rr_2}{R + r_2} = 10 + \frac{5 \times 15}{5 + 15} = \frac{55}{4}\,\Omega$$

$$\therefore \text{Current,} \quad i' = \frac{e_1}{\text{Net resistance}} = \frac{4 \times 10^{-3}}{55/4} = \frac{16}{55} \times 10^{-3}\ \text{A}$$

$$\therefore \text{Current through } R, \quad i_1' = \left(\frac{r_2}{R + r_2}\right) i'$$

$$= \left(\frac{15}{15 + 5}\right)\left(\frac{16}{55}\right) \times 10^{-3}\ \text{A} = \frac{12}{55}\ \text{mA}$$

From superposition principle net current through 5 Ω resistor is

$$i_1' - i_1 = \frac{8}{55}\ \text{mA from } d \text{ to } c$$

Induction of field

The modern view of electromagnetic induction states that induction occurs whether or not a conducting wire or material medium is present.

In general case, Faraday's law can be stated as, "An electric field is induced in any region of space in which a magnetic field is changing with time."

This law is also true if magnetic and electric fields are interchanged, *i.e.*,

"A magnetic field is induced in any region of space in which an electric field is changing with time".

In each case the strength of the induced field is proportional to the rate of change of the inducing field. Induced electric and magnetic fields are at right angles to each other.

Induced electric field

As per Faraday's law, due to change in magnetic field, an electric field is induced.

Let us understand this by considering a ring of radius r. A uniform magnetic field B is perpendicular to its plane.

So, time rate of change of magnetic field is $\frac{dB}{dt}$, then the flux associated will be ϕ. And the induced emf will be $e = \frac{-d\phi_B}{dt}$.

Hence, an induced current will flow in anti-clockwise direction, which means, if there is a current present in the ring, then there must be an electric field. And this electric field is produced by the changing magnetic field.

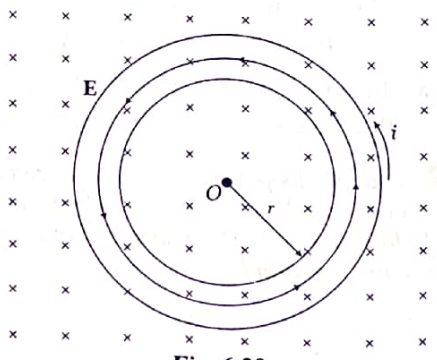


Fig. 6.28

Suppose a particle of charge q_0 is moving around the ring in a circular path. So, the work done by the induced electric field in one revolution is given by

$W = q_0 e$, where e is the induced emf ...(i)

We also know,

$$W = \oint \mathbf{F} \cdot d\mathbf{l} = q_0 \cdot \oint \mathbf{E} \cdot d\mathbf{l} \quad \ldots (ii)$$

Equating Eqs. (i) and (ii), we get

$$q_0 e = q_0 \oint \mathbf{E} \cdot d\mathbf{l} \Rightarrow \quad e = \oint \mathbf{E} \cdot d\mathbf{l} \quad \ldots (iii)$$

Eq. (iii) is the relationship between induced emf and induced electric field.

From the Faraday's law, $e = \dfrac{-d\phi_B}{dt}$ ... (iv)

From Eqs. (iii) and (iv), we get

$$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\phi_B}{dt} \quad \ldots (v)$$

This is known as integral form of Faraday's law of electromagnetic induction.

Note In case of electromagnetic induction line intergral of induced emf **E** around a close path is not zero, *i.e.*, induced electric field is not conservative. In such a field, work done in moving a charge round or close path is not zero.

Example 6.31 *A uniform magnetic field exists in a circular region of radius R centred at O. The field is perpendicular to the plane of paper and its strength varies with time as $B = B_0 t$. Find the induced electric field at a distance r from the centre for (i) $r < R$, (ii) $r > R$. Also, plot a graph between $|E|$ and r for both the cases.*

Sol. As we know, induced electric field is given by

$$\int \mathbf{E} \cdot d\mathbf{l} = \frac{-d\phi_B}{dt}$$

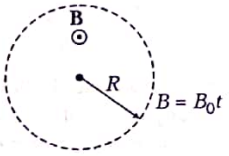
(i) For, $r < R$. Consider a ring of radius $r(< R)$ centered at O. The induced field at the periphery of the ring is circular such that,

$$\oint \mathbf{E} \cdot d\mathbf{l} = E \times 2\pi r \quad \ldots (i)$$

($\because dl$ = circumference of the ring $= 2\pi r$)

As, magnetic flux, $\phi_B = \mathbf{B} \cdot \mathbf{S} = BS \cos\theta$

Here, $\theta = 0°$; so $\phi_B = (B_0 t)\pi r^2$ [$\because B = B_0 t$]



$$\therefore \quad \frac{d\phi_B}{dt} = B_0 \pi r^2 \quad \ldots (ii)$$

Equating Eqs. (i) and (ii), we get

$$\therefore \quad E \times 2\pi r = -B_0 \pi r^2$$

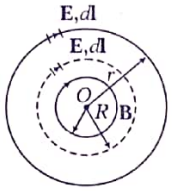
$$E = \frac{-B_2 r}{2} \Rightarrow E \propto r$$

(the sign comes, when clockwise electric field lines are developed)

(ii) For, $r > R$. Consider a ring of radius $r(> R)$ centred at O. Again following the same procedure,

$$\oint \mathbf{E} \cdot d\mathbf{l} = E \times 2\pi r \quad \ldots (iii)$$

Similarly, flux, $\phi_B = (B_0 t)\pi R^2$ (as field is present only in region $0 < r < R$). ... (iv)



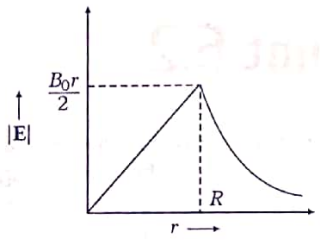
$$\Rightarrow \quad \frac{d\phi_B}{dt} = B_0 \pi R^2$$

Equating Eqs. (iii) and (iv), we get

$$\Rightarrow \quad E 2\pi r = -B_0 \pi R^2$$

$$\Rightarrow \quad E = \frac{-B_0 R^2}{2r} \Rightarrow E \propto \frac{1}{r}$$

Hence, plot between $|\mathbf{E}|$ and r will be



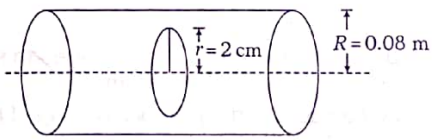
Example 6.32 *The current in an ideal, long solenoid is varied at a uniform rate of* 0.02 A/s. *The solenoid has* 1000 *turns/s and its radius is* 8 *cm.*

(i) *Consider a circle of radius* 2 *cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in* 4 *s.*

(ii) *Find the electric field induced at a point on the circumference of the circle.*

(iii) *Find the electric field induced at a point outside the solenoid at a distance* 9 *cm from its axis.*

Sol.



Given, for solenoid, $n = 1000$ m/s

and $\quad \dfrac{di}{dt} = 0.02$ A/s

$$\Rightarrow \quad di = 0.02\, dt \quad \ldots (i)$$

(i) Magnetic field due to solenoid is given as

$$B = \mu_0 n i$$

Flux through circle of radius r, $\phi_B = B\pi r^2$

($\because \phi_B = \mathbf{B} \cdot \mathbf{S}$)

$$= \mu_0 n i \pi r^2$$

$$\Rightarrow \quad d\phi_B = \mu_0 n \pi r^2 i$$

$$= \mu_0 n \pi r^2 (0.02) dt \quad [\because \text{using Eq. (i)}]$$

$$\Rightarrow \quad \phi = \mu_0 n\pi r^2 (0.02)\int_0^4 dt$$

$$= 4\pi \times 10^{-7} \times 1000 \times \pi(0.02)^2 (0.02 \times 4)$$

$$= 126 \times 10^{-7} \text{ Wb}$$

(ii) Electric field induced is related to emf as, $\oint \mathbf{E} \cdot d\mathbf{l} = \frac{d\phi}{dt}$

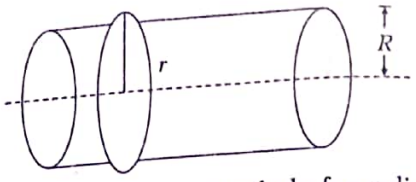
$$\Rightarrow \quad E \cdot 2\pi r = \mu_0 n\pi r^2 \frac{di}{dt}$$

$$E = \frac{\mu_0 nr}{2} \frac{di}{dt}$$

$$= \frac{4\pi \times 10^{-7} \times 1000 \times 0.02}{2} \times 0.02$$

$$= 8\pi \times 10^{-8} \text{ V/m}$$

(iii)



As discussed in (i), similarly for radius outside the solenoid,

$$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{d}{dt}(\mu_0 ni\, \pi R^2)$$

$$\Rightarrow \quad E \cdot 2\pi r = \mu_0 n \frac{di}{dt} \pi R^2 \quad \Rightarrow \quad E = \frac{\mu_0 nR^2}{2r} \frac{di}{dt}$$

$$= \frac{4\pi \times 10^{-7} \times 1000 \times (0.08)^2 \times 0.02}{2 \times 0.09}$$

$$= 28\pi \times 10^{-8} \text{ V/m}$$

Check point 6.2

1. A conducting rod of length l is falling with a constant velocity v perpendicular to a uniform horizontal magnetic field B. A potential difference between its two ends will be
(a) $2Blv$ (b) Blv
(c) $\frac{1}{2}Blv$ (d) $B^2 l^2 v^2$

2. A wire of length 50 cm moves with a velocity of 300 m/min, perpendicular to a magnetic field. If the emf induced in the wire is 2 V, then the magnitude of the field in tesla is
(a) 2 (b) 5
(c) 0.8 (d) 2.5

3. A 10 m wire kept in east-west direction is falling with velocity 5 m/s perpendicular to the field 0.3×10^4 Wb/m^2. The induced emf across the terminal will be
(a) 0.15 V (b) 1.5 mV
(c) 1.5 V (d) 15.0 V

4. A boat is moving due east in a region, where the earth's magnetic field is 5.0×10^{-5} NA^{-1}m^{-1} due north and horizontal. The boat carried a vertical areal 2 m long. If the speed of the boat is 1.50 ms^{-1}, then the magnitude of the induced emf in the wire of areal is
(a) 1 mV (b) 0.75 mV
(c) 0.50 mV (d) 0.15 mV

5. The magnitude of the earth's magnetic field at a place is B_0 and the angle of dip is δ. A horizontal conductor of length l lying along the magnetic north-south moves eastwards with a velocity v. The emf induced across the conductor is
(a) zero (b) $B_0 lv \sin\delta$
(c) $B_0 lv$ (d) $B_0 lv \cos\delta$

6. A conducting circular loop is placed in a uniform magnetic field of induction B tesla with its plane normal to the field. Now, the radius of the loop starts shrinking at the rate (dr/dt). Then, the induced emf at the instant when the radius is r, is
(a) $\pi rB(dr/dt)$ (b) $2\pi rB(dr/dt)$
(c) $\pi r^2 (dB/dt)$ (d) $(\pi r^2/2)B(dr/dt)$

7. Two similar circular loops carry equal currents in the same direction. On moving the coils further apart, the electric current will
(a) remain unaltered
(b) increases in one and decreases in the second
(c) increase in both
(d) decrease in both

8. A circular coil of mean radius of 7 cm and having 4000 turns is rotated at the rate of 1800 rev/min in the earth's magnetic field (B = 0.5 gauss), the maximum emf induced in coil will be
(a) 1.158 V (b) 0.57 V
(c) 0.29 V (d) 5.8 V

9. A coil of N turns and mean cross-sectional area A is rotating with uniform angular velocity ω about an axis at right angle to uniform magnetic field B. The induced emf E in the coil will be
(a) $NBA\sin\omega t$
(b) $NB\omega\sin\omega t$
(c) $NB/A\sin\omega t$
(d) $NBA\omega\sin\omega t$

10. A 0.1 m long conductor carrying a current of 50 A is held perpendicular to a magnetic field of 1.25 mT. The mechanical power required to move the conductor with a speed of 1 ms^{-1} is
(a) 62.5 mW (b) 625 mW
(c) 6.25 mW (d) 12.5 mW
(e) 125 mW

11. A metal rod of length 2 m is rotating with an angular velocity of 100 rads^{-1} in a plane perpendicular to a uniform magnetic field of 0.3 T. The potential difference between the ends of the rod is

(a) 30 V (b) 40 V
(c) 60 V (d) 600 V

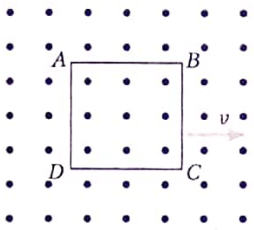
12. A conducting rod of length l is moving in a transverse magnetic field of strength B with velocity v. The resistance of the rod is R. The current in the rod is

(a) $\frac{Blv}{R}$ (b) Blv
(c) zero (d) $\frac{B^2v^2l^2}{R}$

13. A rectangular coil rotates about an axis normal to the magnetic field. If E_m is the maximum value of the induced emf, then the instantaneous emf when the plane of the coil makes an angle of 45° with the magnetic field is

(a) $\frac{1}{2}E_m$ (b) $\frac{1}{4}E_m$
(c) $\frac{1}{\sqrt{2}}E_m$ (d) E_m

14. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced



(a) in AD but not in BC (b) in BC but not in AD
(c) Neither in AD nor in BC (d) in both AD and BC

15. Consider the following statements.

A. An emf can be induced by moving a conductor in a magnetic field.

B. An emf can be induced by changing the magnetic field.

(a) Both A and B are true (b) A is true but B is false
(c) B is true but A is false (d) Both A and B are false

3 Self-induction

Self-induction is the property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an emf in itself.

This induced emf is also called **back emf**. When the current in a coil is switched on, then the self-induction opposes the growth of the current and when it is switched off, then the self-induction opposes the decay of the current. Hence, self-induction is also known as **inertia of electricity.**

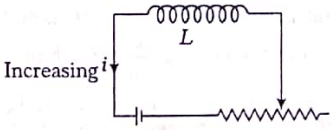


Fig. 6.29

A circuit or part of a circuit, that is designed to have a particular induction is called an inductor. The usual symbol for an inductor is shown in Fig. 6.30.

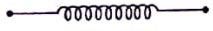


Fig. 6.30

Thus, an inductor is a circuit element which opposes the change in current through it. It may be a circular coil, solenoid, etc.

Coefficient of self-induction

Definition using magnetic flux

If i is the strength of current flowing through a coil at any time and ϕ is the amount of magnetic flux linked with all the turns of the coil at that time, then it is found that

$$\phi \propto i \text{ or } \phi = Li \Rightarrow L = \frac{\phi}{i}$$

where, L is a constant of proportionality and is called **coefficient of self-induction** or **self-inductance of the coil**. The value of L depends on number of turns N, area of cross-section A and nature of material of the core on which coil is wound.

For N number of turns in coil, $L = \frac{N\phi}{i}$.

Definition using induced emf

If a current i is passed in a circuit and it is changed with a rate di/dt, then the induced emf e produced in the circuit is directly proportional to the time rate of change of current. Thus,

$$e \propto \frac{di}{dt}$$

When the proportionality sign is removed, the same constant L again comes here.

Hence, $$e = -L\frac{di}{dt}$$

The minus sign here is a reflection of Lenz's law. It says that the self-induced emf in a circuit opposes any change in the current in that circuit.

Average induced emf e is given by

$$e = -L\frac{di}{dt} = -\frac{L(i_2 - i_1)}{\Delta t}.$$

From the above equation, we can write

$$L = \left|\frac{-e}{di/dt}\right| \quad \text{...(i)}$$

If $\frac{di}{dt} = 1$, then $L = e$

So, **the coefficient of self-induction** or **self-inductance of a coil is equal to the emf induced in the coil when time rate of change of current through the coil is unity.** Inductance is a scalar quantity.

Some important points regarding coefficient of self-induction

(i) The SI unit of L is henry.

(ii) According to Eq. (i), we can write

$$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere/second}} = \frac{1 \text{ volt-second}}{\text{ampere}}$$

Self-inductance of a coil is said to be one henry (H) when a current change at the rate of 1 ampere/second through the coil induces an emf of 1 volt in the coil.

(iii) Dimensions of self-inductance L is $[ML^2T^{-2}A^{-2}]$.

Kirchhoff's second law with an inductor

Let us consider a circuit as shown in Fig. 6.31.

According to Kirchhoff's second law (loop rule), when we go through an inductor in the same direction as the assumed current, we encounter a voltage drop equal to $L\,di/dt$, where di/dt is to be substituted with sign.

e.g., in the loop shown in figure, Kirchhoff's second law gives the equation,

$$\Rightarrow \quad E - iR - L\frac{di}{dt} = 0$$

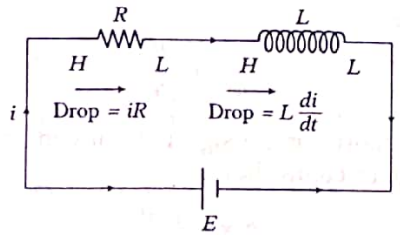
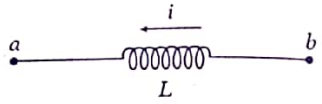


Fig. 6.31

Example 6.33 *The inductor shown in figure has inductance 0.54 H and carries a current in the direction shown that is decreasing at a uniform rate* $\frac{di}{dt} = -0.03$ *A/s.*



(i) Find the self-induced emf.

(ii) Which end of the inductor a or b is at a higher potential?

Sol. (i) Given, inductance, $L = 0.54$ H

$$\frac{di}{dt} = -0.03$$

Self-induced emf is given as

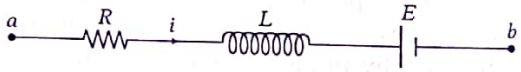
$$e = -L\frac{di}{dt} = (-0.54)(-0.03) \text{ V}$$

$$= 1.62 \times 10^{-2} \text{ V}$$

(ii) We know that, $V_{ba} = L\frac{di}{dt} = -1.62 \times 10^{-2}$ V

Since, $V_{ba}\,(= V_b - V_a)$ is negative. It implies that $V_a > V_b$ or a is at higher potential.

Example 6.34 *In the circuit diagram shown in figure,* $R = 10\,\Omega$, $L = 5$ *H,* $E = 20$ *V,* $i = 2$ *A. This current is decreasing at a rate of* -1.0 *A/s. Find* V_{ab} *at this instant.*



Sol. Potential difference across inductor is given as

$$V_L = L\frac{di}{dt} = (5)(-1.0) = -5 \text{ V}$$

Now, using Kirchhoff's second law, $V_a - iR - V_L - E = V_b$

$$\therefore \quad V_{ab} = V_a - V_b = E + iR + V_L$$

$$= 20 + (2)(10) - 5 = 35 \text{ V}$$

Note In the above example, as the current is decreasing, the inductor can be replaced by a source of emf $e = \left|L \cdot \frac{di}{dt}\right| = 5$ V in such a manner that this emf supports the decreasing current, or it sends the current in the circuit in the same direction as the existing current. So, positive terminal of this source is towards b. Thus, the given circuit can be drawn as

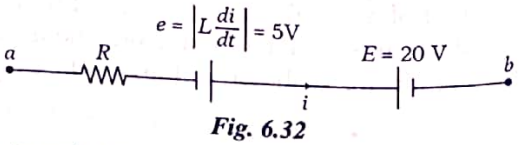
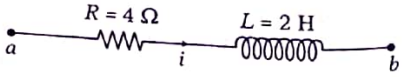


Fig. 6.32

Now, we can find V_{ab}.

Example 6.35 *In the figure shown* $i = 10\,e^{-4t}$ *A. Find* V_L *and* V_{ab}.



Sol. Potential difference across the inductor is given as

$$V_L = +L\frac{di}{dt} = (2)\frac{d}{dt}(10e^{-4t}) = -80\,e^{-4t}$$

Further, by using Kirchhoff's second law,

$$V_a - iR - L\frac{di}{dt} = V_b$$

$$\therefore\ V_a - V_b = iR + L\frac{di}{dt} \quad \text{or} \quad V_{ab} = (10e^{-4t})(4) - 80e^{-4t} = -40e^{-4}t$$

Self-inductance of a coil

Consider a coil of radius r and current i is flowing through the coil. If number of turns in the coil is N, then magnetic field at the centre is given as

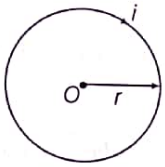


Fig. 6.33

$$\Rightarrow \qquad B_{\text{centre}} = \frac{\mu_0 Ni}{2r} \quad \text{and} \quad \phi_B = BS$$

where, S = area of cross-sectional of the coil = πr^2

So, $$\phi_B = \frac{\mu_0 Ni}{2r} \times \pi r^2 \Rightarrow \phi_B = \frac{\mu_0 \pi Nir}{2}$$

Now, net flux for the coil of N number of turns,

$$\Rightarrow \qquad \phi_B = N\phi_0 = \frac{N \times \mu_0 \pi Nir}{2} = \frac{\mu_0 \pi N^2 ir}{2}$$

We know that, $L = \dfrac{\phi_B}{i} \Rightarrow \boxed{L = \dfrac{\mu_0 N^2 \pi r}{2}\ \text{henry}}$

Self-inductance of a solenoid

Let us find the inductance of a uniformly wound solenoid having N turns and length l. Assume that l is much longer than the radius of the windings and that the core of the solenoid is air. We can assume that the interior magnetic field due to a current 'i' is uniform and given by equation,

$$\Rightarrow \qquad B = \mu_0 ni = \mu_0\left(\frac{N}{l}\right)i$$

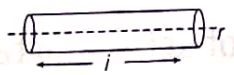


Fig. 6.34

where, $n = \dfrac{N}{l}$ is the number of turns per unit length.

The magnetic flux through each turn is

$$\phi_B = BS = \mu_0 \frac{NS}{l} i$$

Here, S is the cross-sectional area of the solenoid.

Now, as we know, $L = \dfrac{N\phi_B}{i} = \dfrac{N}{i}\left(\dfrac{\mu_0 NSi}{l}\right) = \dfrac{\mu_0 N^2 S}{l}$

$$\Rightarrow \qquad \boxed{L = \frac{\mu_0 N^2 S}{l}}$$

This result shows that L depends on dimensions (S, l) and is proportional to the square of the number of turns.

$$\therefore \qquad L \propto N^2$$

Because $N = nl$, we can also express the result in the form,

$$L = \mu_0 \frac{(nl)^2}{l} S = \mu_0 n^2 Sl = \mu_0 n^2 V \quad \text{or} \quad L = \mu_0 n^2 V$$

Here, $V = Sl$ is the volume of the solenoid.

Note If the space inside the solenoid is filled with a material of relative permeability μ_r, then $L = \mu_0 \mu_r n^2 Sl$.

Example 6.36 *(i) Calculate the inductance of an air core solenoid containing 300 turns, if the length of the solenoid is 25 cm and its cross-sectional area is 4 cm^2.*
(ii) Calculate the self-induced emf in the solenoid, if the current through it is decreasing at the rate of 50 A/s.

Sol. (i) The inductance of a solenoid is given by

$$L = \frac{\mu_0 N^2 S}{l}$$

where, S is the cross-sectional area of solenoid.
Substituting the values, we have

$$L = \frac{(4\pi \times 10^{-7})(300)^2 (4 \times 10^{-4})}{(25 \times 10^{-2})}\text{H}$$

$$= 1.81 \times 10^{-4}\ \text{H}$$

(ii) The self-induced emf is given by

$$e = -L\frac{di}{dt}$$

Here, $\dfrac{di}{dt} = -50\,\text{As}^{-1}$

$$\therefore \qquad e = -(1.81 \times 10^{-4})(-50) = 9.05 \times 10^{-3}\ \text{V}$$

or $\qquad e = 9.05\ \text{mV}$

Energy stored in an inductor

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.

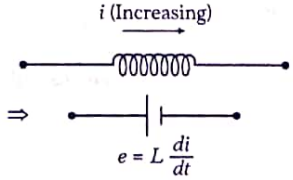


Fig. 6.35

An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power,

$$P = \frac{dW}{dt} = -ei = -Li\frac{di}{dt}$$

From, $dW = -dU$ or $\frac{dW}{dt} = -\frac{dU}{dt}$, we have

$$\frac{dU}{dt} = Li\frac{di}{dt}$$

or $$dU = Li\, di$$

The total energy U supplied while the current increases from zero to a final value i is

$$U = L\int_0^i idi = \frac{1}{2}Li^2$$

$$\therefore \quad \boxed{W = U = \frac{1}{2}Li^2}$$

Energy stored per unit volume in magnetic field is known as energy density.

$$\therefore \quad \boxed{\text{Energy density} = \frac{U}{V} = \frac{1}{2}\frac{B^2}{\mu_0}}$$

Thus, if $i = 1\,A$, then $2W = L$

Hence, **the coefficient of self-inductance is equal to twice the work done in establishing a flow of one ampere current in the circuit.**

Note Time constant in R-L circuit is $\tau = \frac{L}{R}$.

Example 6.37 *What inductance would be needed to store $1\,kWh$ of energy in a coil carrying a $200\,A$ current?* $(1\,kWh = 3.6\times 10^6\,J)$

Sol. We have, $i = 200$ A

and $U = 1$ kWh $= 3.6\times 10^6$ J

$$\therefore \text{ Self-inductance, } L = \frac{2U}{i^2} = \frac{2(3.6\times 10^6)}{(200)^2} = 180\text{ H}$$

$$\left(\because U = \frac{1}{2}Li^2\right)$$

Example 6.38 *Two coils having self-inductances, $L_1 = 5\,mH$ and $L_2 = 1\,mH$. The current in the coil is increasing at same constant rate at a certain instant and the power supplied to the coils is also same. Find the ratio of*

(i) induced voltage

(ii) current

(iii) energy stored in two coils at that instant

Sol. Given, $L_1 = 5$ mH and $L_2 = 1$ mH

(i) As we know, induced voltage is given by, $e = \frac{Ldi}{dt}$

$$\Rightarrow \quad \frac{e_1}{e_2} = \frac{L_1(di/dt)}{L_2(di/dt)} = \frac{L_1}{L_2} = \frac{5}{1} = 5:1 \quad \text{...(i)}$$

(ii) Power in the coil is given by

$$P = ei$$

Here, $$P_1 = P_2 \Rightarrow e_1 i_1 = e_2 i_2 \Rightarrow \frac{i_1}{i_2} = \frac{e_2}{e_1}$$

Using Eq. (i), we can write, $\frac{i_1}{i_2} = \frac{1}{5} = 1:5$

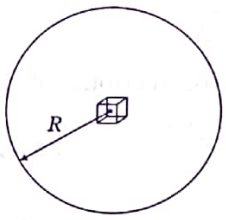
(iii) Energy stored in a coil is given by

$$U = \frac{1}{2}Li^2$$

$$\Rightarrow \quad \frac{U_1}{U_2} = \frac{(1/2)L_1 i_1^2}{(1/2)L_2 i_2^2} = \frac{L_1}{L_2}\left(\frac{e_2}{e_1}\right)^2 = \frac{5}{1}\left(\frac{1}{5}\right)^2 = 1:5$$

Example 6.39 *Suppose a cube of volume $2mm^3$ is placed at the centre of a circular loop of radius $5cm$ carrying current $2A$. Find the magnetic energy stored inside the cube.*

Sol.



Magnetic field at the centre of the circular loop is given by $B = \frac{\mu_0 i}{2R}$

We know, energy density, $\mu = \frac{B^2}{2\mu_0}$

and energy stored in the cube will be given by

$$U = \mu V_0 = \frac{B^2}{2\mu_0}V_0$$

where, V_0 is the volume of the cube.

Substituting the values in the above equation,

$$= \frac{1}{2\mu_0}\times\left(\frac{\mu_0 i}{2R}\right)^2 V_0 = \frac{\mu_0 i^2 V_0}{8R^2}$$

$$= \frac{4\pi\times 10^{-7}\times 2^2\times 2\times 10^{-9}}{8\times(0.05)^2} = 16\,\pi\times 10^{-14}\text{ J}$$

Combination of self-inductances

(i) In series

If several inductances are connected in series and there is interactions through self-inductance only as shown in Fig. 6.36,

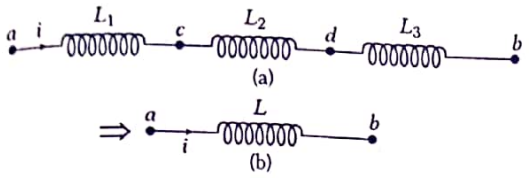


Fig. 6.36

Then their equivalent inductance is given by

Equivalent inductance, $L = L_1 + L_2 + L_3$

(ii) In parallel

When several inductances are connected in parallel as shown in the following figure 6.37, then their equivalent inductance can be calculated as

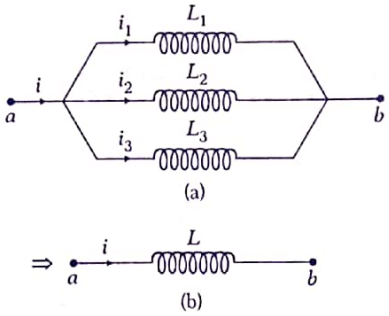


Fig. 6.37

The equivalent inductance L is given by

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Check point 6.3

1. The SI unit of inductance, the henry, can be written as
(a) weber/ampere (b) volt-second/ampere
(c) joule/(ampere)^2 (d) ohm-second

2. A long solenoid has 500 turns. When a current of 2 A is passed through it, then the resulting magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is
(a) 1.0 H (b) 4.0 H (c) 2.5 H (d) 2.0 H

3. If a current of 10 A changes in one second through a coil, and the induced emf is 10 V,then the self-inductance of the coil is
(a) $\frac{2}{5}$ H (b) $\frac{4}{5}$ H
(c) $\frac{5}{4}$ H (d) 1 H

4. During a current change from 2 A to 4 A in 0.5 s, 8 V of emf is developed in a coil. The coefficient of self-induction is
(a) 1 H (b) 2 H (c) 4 H (d) 8 H

5. The current passing through a choke coil of 5 H is decreasing at the rate of 2 A/s. The emf developing across the coil is
(a) 10 V (b) –10 V
(c) 2.5 V (d) –2.5 mV

6. In a coil of self-inductance 0.5 H, the current varies at a constant rate from zero to 10 A in 2 s. The emf generated in the coil is
(a) 10 V (b) 5 V
(c) 2.5 V (d) 1.25 V

7. The self-inductance of a long solenoid cannot be increased by
(a) increasing its area of cross-section
(b) increasing its length
(c) changing the medium with greater permeability
(d) increasing the current through it

8. Self-inductance of a coil is 50 mH. A current of 1 A passing through the coil reduces to zero at steady rate in 0.1 s, the self-induced emf is
(a) 5 V (b) 0.05 V (c) 50 V (d) 0.5 V

9. The self-inductance of a coil is L. Keeping the length and area same, the number of turns in the coil is increased to four times. The self-inductance of the coil will now be
(a) $\frac{1}{4}L$ (b) L (c) $4L$ (d) $16L$

10. In circular coil, when number of turns is doubled and resistance becomes (1/4)th of initial, then inductance becomes
(a) 4 times (b) 2 times (c) 8 times (d) No change

11. The self-inductance of solenoid of length L, area of cross-section S and having N turns is
(a) $\frac{\mu_0 N^2 S}{L}$ (b) $\frac{\mu_0 NS}{L}$ (c) $\mu_0 N^2 LS$ (d) $\mu_0 NLS$

12. A solenoid has 2000 turns would over a length of 0.30 m. The area of its cross-section is 1.2×10^{-3}m^2. If an initial current of 2 A in the solenoid is reversed in 0.25 s, then the emf induced in the coil is
(a) 6×10^{-4} V (b) 4.8×10^{-3} V
(c) 6×10^{-2} V (d) 32.1×10^{-2} V

13. A 50 mH coil carries a current of 2 A. The energy stored in joules is
(a) 1 (b) 0.1 (c) 0.05 (d) 0.5

14. In an inductor of inductance $L = 100$ mH, a current of $I = 10$ A is flowing. The energy stored in the inductor is
(a) 5 J (b) 10 J (c) 100 J (d) 1000 J

15. Two pure inductors each of self-inductance L are connected in parallel but are well separated from each other. The total inductance is
(a) $2L$ (b) L (c) $\frac{L}{2}$ (d) $\frac{L}{4}$

4 Mutual induction

Consider two neighbouring coils of wire as shown in Fig. 6.38. A current flowing in coil 1 produces magnetic field and hence a magnetic flux through coil 2. If the current in coil 1 changes, then the flux through coil 2 changes as well. According to Faraday's law, this induces an emf in coil 2. In this way, a change in the current in one circuit can induce a current in a second circuit. This phenomenon is known as **mutual induction.**

In other words, the phenomena according to which an opposing emf is produced in a coil as a result of change in current a magnetic flux linked with a neighbouring coil is called mutual induction.

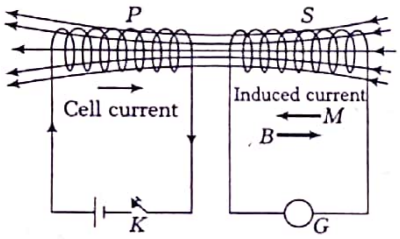


Fig. 6.38

Like the self-inductance (L), two circuits has mutual inductance (M).

Coefficient of mutual inductance

Definition using magnetic flux

Suppose the circuit 1 has a current i_1 flowing in it. Then, total flux $N_2\phi_{B_2}$ linked with circuit 2 is proportional to the current in circuit 1.

Thus, $N_2\,\phi_{B_2} \propto i_1$ or $N_2\phi_{B_2} = Mi_1$

Here, the proportionality constant M is known as coefficient of mutual inductance 'M' of the two circuits.

Thus, $$M = \frac{N_2\phi_{B_2}}{i_1}$$

From this expression M can be defined as **the total flux $N_2\phi_{B_2}$ linked with circuit 2 per unit current in circuit 1.**

Definition using induced emf

If we change the current in circuit 1 at a rate di_1/dt, an induced emf e_2 is developed in circuit 1, which is proportional to the rate di_1/dt.

Thus, $e_2 \propto di_1/dt$ or $e_2 = -Mdi_1/dt$ ...(i)

Here, the proportionality constant is again M. Minus sign indicates that e_2 is in such a direction that it opposes any change in the current in circuit 1. From the above equation, we can write

$$M = \left|\frac{-e_2}{di_1/dt}\right|$$

If $\frac{di}{dt} = 1$, then $M = e$. Thus, coefficient of mutual induction or mutual inductance of two coils is equal to the emf induced in one coil when rate time of change of current through the other coil is unity.

Some important points regarding the coefficient of mutual inductance

(i) The SI unit of coefficient of mutual inductance is **henry** (1H).

Coefficient of mutual induction or mutual inductance of two coils is said to be **one henry**, when a current change at the rate of one ampere/second in one coil induces an emf of one volt in the other coil.

(ii) M depends upon closeness of the two circuits, their orientations and sizes and the number of turns, etc.

(iii) Its dimensions are $[ML^2T^{-2}A^{-2}]$.

(iv) **Reciprocity theorem** This theorem states that mutual inductance due to secondary coil on primary coil is equal to the mutual induction due to primary on secondary coil, *i.e.*, $M_{21} = M_{12} = M$

Using Eq. (i),

$$e_2 = -M(di_1/dt)$$

and $$e_1 = -M(di_2/dt)$$

$$\Rightarrow \quad M_{12} = \frac{N_2\phi_{B_2}}{i_1}$$

and $$M_{21} = \frac{N_1\phi_{B_1}}{i_2}$$

(v) Average induced emf, $\bar{e} = -M\frac{\Delta i}{\Delta t} = -\frac{-M(i_2 - i_1)}{\Delta t}$.

(vi) *A good approach for calculating the mutual inductance of two circuits consists of the following steps*

(a) Assume anyone of the circuits as primary (first) and the other as secondary (second).

(b) Suppose a current i_1 flows through the primary circuit.

(c) Determine the magnetic field **B** produced by the current i_1.

(d) Obtain the magnetic flux ϕ_{B_2}.

(e) With the flux known, the mutual inductance can be found from

$$M = \frac{N_2\phi_{B_2}}{i_1}$$

Mutual inductance for two concentric coils

Let us consider a coil having N_1 turns and of radius r is surrounded by another coil having N_2 turns and of radius R as shown in the Fig. 6.39 ($R >> r$).

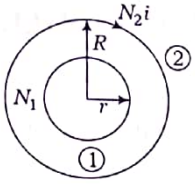


Fig. 6.39

Magnetic field due to coil 2 at centre, $B_2 = \frac{\mu_0 N_2 i}{2R}$

Flux through coil 1, $\phi_{B_1} = N_1 B_2 S = \frac{N_1 \mu_0 N_2 i}{2R} \pi r^2 = Mi$

$$\Rightarrow \quad \boxed{M = \frac{\mu_0 N_1 N_2 \pi r^2}{2R}}$$

Mutual inductance of two long co-axial solenoids

Fig. 6.40 shows two long co-axial solenoids, each of length l. Let n be the number of turns per unit length of inner solenoid S_1 of radius r_1 and n_1 is number of turns per unit length of outer solenoid S_2 of radius r_2.

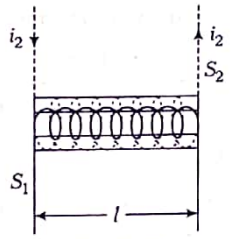


Fig. 6.40

Magnetic field due to S_1, $B_1 = \mu_0 n_1 i$

Flux through S_2,

$$\phi_B = (n_2 l) B_1 \pi r_1^2$$

$$= (n_2 l) \mu_0 n_1 i \pi r_1^2 = Mi$$

$$\Rightarrow \quad M = \mu_0 n_1 n_2 \pi r_1^2 l$$

$$\boxed{M = \frac{\mu_0 N_1 N_2 \times S}{l}}$$

where, S is the area of cross-section of inner solenoid.

Note We do not take the area of cross-section of outer solenoid because there is no magnetic field between the two solenoids.

Mutual inductance of a solenoid surrounded by a coil

Let N_1 be the number of turns of solenoid of radius r_1 and N_2 be the number of turns of coil of radius r_2.

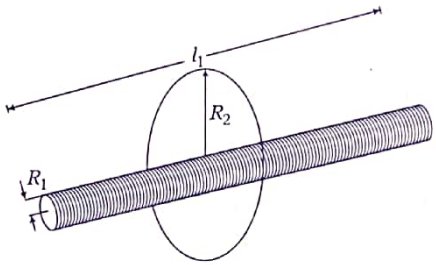


Fig. 6.41

Magnetic field due to S,

$$B = \mu_0 (N_1 / l_1) i$$

Then, flux through coil is

$$\phi_B = N_2 BS = N_2 \mu_0 \frac{N_1}{l} i \pi R_1^2 = Mi$$

$$\Rightarrow \quad \boxed{M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l}} \qquad \left(\because M = \frac{N_2 \phi_B}{i} \right)$$

From the above relation, we see that, M is independent of radius R_2. This is because, solenoid's magnetic field is confined to its interior.

Note Mutual inductance of a small circular loop and a rectangular loop ($r << a, b$)

$$\boxed{M = \frac{\mu_0}{4\pi} \frac{8\sqrt{a^2 + b^2}}{ab} \pi r^2}$$

Mutual inductance between two coils depends upon the geometry of two coils and their geometrical orientation with respect to each other. To find mutual inductance for a given arrangement, we assume a current flowing through one of the coils and find the flux through the other coil. Then, using the formula $\phi = MI$, mutual inductance M can be calculated.

Coefficient of coupling

The coefficient of coupling of two coils gives a measure of the manner in which the two coils are coupled together. If L_1 and L_2 are the self-inductances of two coils and M is their mutual inductance, then their coefficient of coupling is given by

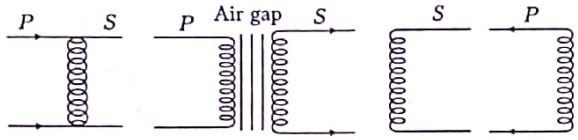
$$K = \frac{M}{\sqrt{L_1 L_2}}$$

It is also defined as

$$K = \frac{\text{magnetic flux linked in secondary coil}}{\text{magnetic flux linked in primary coil}}$$

where, $0 \leq K \leq 1$

So, when $K = 0$, there will be no coupling,
$0 < K < 1$, for loose coupling
and $K = 1$, for maximum coupling. It occurs when the two coils are wound over each other over a ferromagnetic core. The value of K is a number, depending on the geometry of the coils and their relative closeness.



(A) $K = 1$ (B) $0 < K < 1$ (C) $K = 0$

Fig. 6.42

Combination of mutual inductances

(i) **In series** If two coils of self-inductances L_1 and L_2 having mutual inductance M are in series, situated close to each other, then net inductance will be

$$L_S = L_1 + L_2 \pm 2M$$

(ii) **In parallel** If two coils of self-inductances L_1 and L_2 having mutual inductance M, are connected in parallel, situated close to each other, then

$$L_P = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}.$$

Example 6.40 *Calculate the mutual inductance between two coils when a current 2A changes to 6A in 0.2 s and induces an emf of 20mV in the secondary coil.*

Sol. We know that emf is related to the mutual inductance by

$$e = M\frac{\Delta i}{\Delta t}$$

Substituting the given values,

$$20 \times 10^{-3} = \frac{M(6-2)}{0.2} = \frac{M \times 4}{0.2} = 20M$$

$$\Rightarrow \quad M = 10^{-3}\text{H} = 1\text{mH}$$

Example 6.41 *A straight solenoid has 50 turns per cm in primary and 200 turns in the secondary. The area of cross-section of the solenoid is 4 cm^2. Calculate the mutual inductance.*

Sol. The magnetic field at any point inside the straight solenoid of primary with n_1 turns per unit length carrying a current i_1 is given by the relation,

$$B = \mu_0 n_1 i_1$$

The magnetic flux through the secondary of N_2 turns each of area S is given as

$$N_2\phi_2 = N_2(BS) = \mu_0 n_1 N_2 i_1 S$$

$$\therefore \quad M = \frac{N_2\phi_2}{i_1} = \mu_0 n_1 N_2 S$$

Substituting the values,

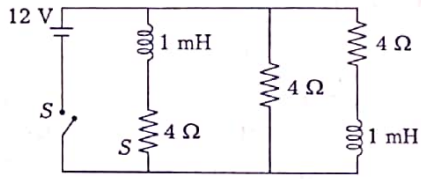
$$M = (4\pi \times 10^{-7})\left(\frac{50}{10^{-2}}\right)(200)(4 \times 10^{-4})$$

$$= 5 \times 10^{-4}\text{ H}$$

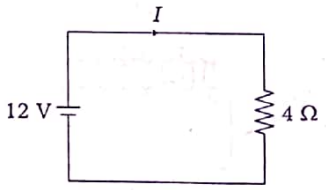
Growth and decay of current in *L-R* circuit

If a circuit containing a pure inductor L and a resistor R in series with a battery and a key is closed, then the current through the circuit rises exponentially and reaches upto a certain maximum value (steady state). If circuit is opened from its steady state condition, then current through the circuit decreases exponentially. In case of growth of current at time $t = 0$, inductor offers infinite resistance and at time $t = \infty$ (steady state) inductor offers zero resistance. This type of circuit is called L-R circuit.

Example 6.42 *Find current through the battery*
(i) just after the switch is closed.
(ii) long after the switch has been closed.

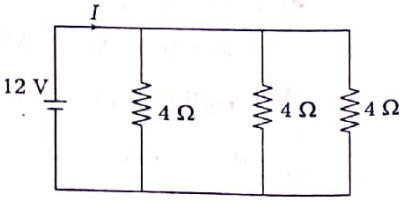


Sol. (i) Just after closing the switch, inductor offers infinite resistance. So, the circuit will become as



So, the current will be $I = \frac{V}{r} = \frac{12}{4} = 3\text{ A}$

(ii) After long time, inductor offers zero resistance. So, the equivalent circuit will be



Net resistance, $R_{eq} = \frac{4 \times 4 \times 4}{4+4+4} = \frac{64}{12} = 5.33$

$\therefore$ Current, $I = \frac{V}{R} = \frac{12}{5.33}$ or $\frac{12 \times 12}{64} = \frac{144}{64} = 2.25\text{ A}$

Growth of current

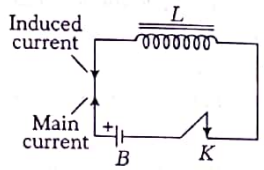
Let us consider a circuit as shown in Fig. 6.43, when the switch K is closed, then the value of current at any instant of time t after closing the circuit (when current is rising) is given by

$$i = i_0\left[1 - e^{-\frac{R}{L}t}\right] \quad \ldots(i)$$

where, $i_0 = i_{max} = \frac{E}{R}$ = steady state current at $t = \infty$

and time constant $\tau_L = L/R$ [time constant]

So, the Eq. (i) can be re-written as $i = i_0(1 - e^{-t/\tau_L})$



Growth of current

Fig. 6.43

Here, τ_L refers at time equal to one time constant the current has risen to $(1 - 1/e)$ or about 63% of its final value i_0. The i-t graph will be shown as Fig. 6.44.

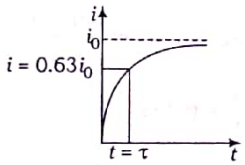


Fig. 6.44

Let us have an insight into the behaviour of an L-R circuit from energy considerations. The instantaneous rate at which the source delivers energy to the circuit ($P = Ei$) is equal to the instantaneous rate at which energy is dissipated in the resistor ($= i^2R$) plus the rate at which energy is stored in the inductor ($= iV_{bc} = Li\, di/dt$)

or $$\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt}$$

Thus, $$Ei = i^2R + Li\frac{di}{dt}$$

Note At any time from Fig. 6.53, $V_R + V_L = E$

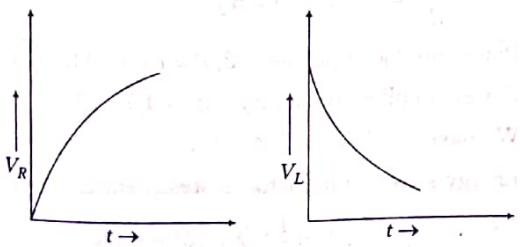


Fig. 6.45

Decay of current

Let us consider the Fig. 6.46, in which now the key K is open. Now, the current will decrease with time.

During the decaying of current, the value of current at any instant of time t after opening from the steady state condition is given by $i = i_0 e^{-\frac{R}{L}t} = i_0 e^{-t/\tau_L}$

Here, τ_L is the time for current to decrease to $1/e$ or about 37% of its original value.

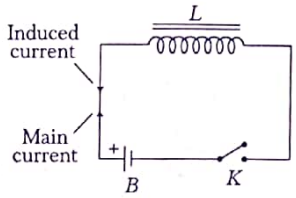


Fig. 6.46 Decay of current

The i-t graph is shown as Fig 6.47.

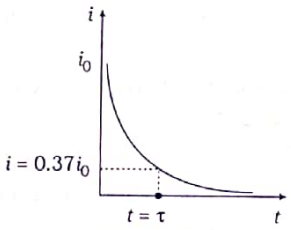


Fig. 6.47

The energy that is needed to maintain the current during this decay is provided by energy stored in the magnetic field. Thus, the rate at which energy is dissipated in the resistor = rate at which the stored energy decreases in magnetic field of inductor. This is given by

$$i^2R = -\frac{dU}{dt} = -\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\left(-\frac{di}{dt}\right) \text{ or } i^2R = Li\left(\frac{-di}{dt}\right)$$

Note At any time, t, $V_R = V_L$

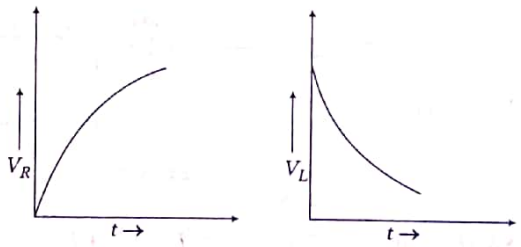


Fig. 6.48

Example 6.43 *A coil of resistance 20 Ω and inductance 0.5 H is switched to DC 200 V supply. Calculate the rate of increase of current*

(i) *at the instant of closing the switch and*

(ii) *after one time constant.*

(iii) *find the steady state current in the circuit.*

Sol. (i) This is the case of growth of current in an L-R circuit. Hence, current at time t is given by

$$i = i_0\,(1 - e^{-t/\tau_L})$$

Rate of increase of current, $\frac{di}{dt} = \frac{i_0}{\tau_L} e^{-t/\tau_L}$

At $t = 0$, $\quad \frac{di}{dt} = \frac{i_0}{\tau_L} = \frac{E/R}{L/R} = \frac{E}{L}$

Substituting the value, we have

$$\frac{di}{dt} = \frac{200}{0.5} = 400 \text{ A/s}$$

(ii) At, $t = \tau_L$, $\frac{di}{dt} = (400)e^{-1} = (0.37)\,(400) = 148 \text{ As}^{-1}$

(iii) The steady state current in the circuit,

$$i_0 = \frac{E}{R} = \frac{200}{20} = 10 \text{ A}$$

Example 6.44 *Self-inductance 0.8×10^{-4} H of a uniformly wound solenoid, having resistance 3 Ω is broken up into two identical coils. Those coils are connected in series across a 6 V battery of negligible resistance. Find time constant and steady state current.*

Sol. For a solenoid, self-inductance is given by

$$L = \frac{\mu_0 N^2 \pi r^2}{l}$$

where, r = radius of solenoid, l = length of solenoid,

N = number of turns

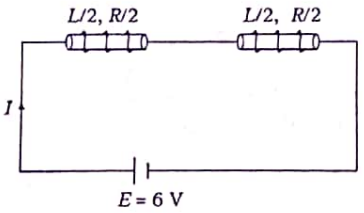
and we know that, resistance is given by

$$R = \rho \frac{l}{A} = \rho \frac{N 2\pi r}{A}$$

So, on breaking it into two parts, now inductance and resistance is given as

$$N' = \frac{N}{2},\ l' = \frac{l}{2} \Rightarrow\ L' = \frac{\mu_0 (N/2)^2 \pi r^2}{l/2} = \frac{L}{2}$$

$$R' = \frac{\rho (N/2) \times 2\pi r}{A} = R$$



So, the equivalent inductance, $L_{eq} = L_1 + L_2$

$$= L/2 + L/2 = L$$

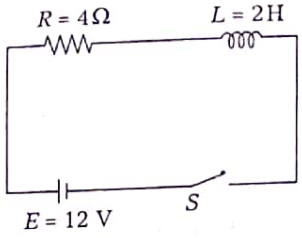
and equivalent resistance, $R_{eq} = R_1 + R_2 = R/2 + R/2 = R$

So, the time constant,

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{0.8 \times 10^{-4}}{3} = 0.267 \times 10^{-4} \text{s}$$

and current, $I = \frac{E}{R_{eq}} = \frac{E}{R} = \frac{6}{3} = 2 \text{A}$

Example 6.45



At $t = 0$, switch S is closed, calculate

(i) initial rate of increase of current, i.e., $\frac{di}{dt}$ at $t = 0$.

(ii) $\frac{di}{dt}$ at time when current in the circuit is 0.5A.

(iii) Current at $t = 0.6$ s.

(iv) rate at which energy of magnetic field is increasing, rate of heat produced in resistance and rate at which energy is supplied by battery when $i = 0.5$A.

(v) energy stored in inductor in steady state.

Sol. We know that

$$E = iR + L\, di/dt$$

$$\frac{di}{dt} = \frac{E - iR}{L}$$

Current in the circuit at any time t,

$$i = i_0(1 - e^{-t/\tau_L})$$

where, $i_0 = E/R$, $\tau_L = L/R$

(i) Rate of increase of current $\frac{di}{dt} = \frac{E - iR}{L}$, when $t = 0$, $i = 0$

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{E}{L} = \frac{12}{2} = 6 \text{ A/s}$$

(ii) At $t = 0.5$ s, $\frac{di}{dt} = \frac{E - iR}{L} = \frac{12 - 0.5 \times 4}{2} = \frac{12 - 2}{2} = 5 \text{ A/s}$

(iii) At $t = 0.6$ s, current $i = \frac{E}{R}(1 - e^{-\frac{Rt}{L}})$

$$= \frac{12}{4}(1 - e^{\frac{4 \times 0.6}{0.5}}) = 3(1 - e^{2.4}) \text{A}$$

(iv) Energy stored in inductor in steady state is given by

$$U = \frac{1}{2} L i^2 \ \Rightarrow\ \frac{dU}{dt} = Li\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{E - iR}{L} = \frac{12 - 0.5 \times 4}{2} = 5 \text{ A/s}$$

$$\frac{dU}{dt} = 2 \times 0.5 \times 5 = 5 \text{ J/s}$$

Power produced per second, $P = i^2 R = (0.5)^2 \times 4 = 1 \text{ J/s}$

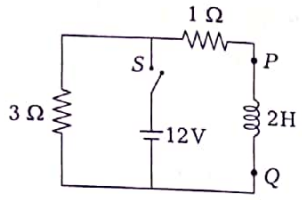
Power supplied by battery $= Ei = 0.5 \times 12 = 6 \text{ J/s}$

(v) We have $i_0 = E/R = 12/4 = 3 \text{ A}$

Energy stored in inductor in steady state,

$$U = \frac{1}{2} L i_0^2 = \frac{1}{2} \times 2 \times (3)^2 = 9 \text{ J/s}$$

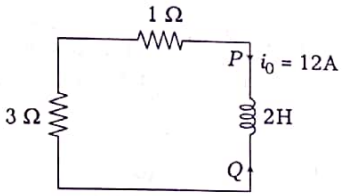
Example 6.46



The switch is closed for a long time and then opened at time $t = 0$. Find the initial voltage across L after $t = 0$, which end is at higher potential P or Q?

Sol. Initially, current will be, $i_0 = \frac{12}{1} = 12\,A$

So, the above circuit would be now as



Current at any time is given by $i = i_0\, e^{-t/\tau_L}$ and $i_0 = 12\,A$

$\because$ $\tau_L = L/R = \frac{2}{(1+3)} = \frac{2}{4} = 0.5$

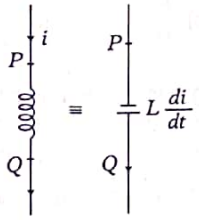
Current, $i = 12e^{-t/0.5}$

$\Rightarrow$ $\frac{di}{dt} = 12e^{-t/0.5}\left(-\frac{1}{0.5}\right) \Rightarrow \left|\frac{di}{dt}\right| = 24e^{-2t}$

So, $V_L = L\frac{di}{dt} = 48e^{-2t}$

At $t = 0, V_L = 48\,V$

i.e., current is decreasing,

$\Rightarrow$ 

$\Rightarrow$ $V_Q > V_P$

Applications of EMI: eddy current

When a changing magnetic flux is applied to a bulk piece of conducting material, then circulating current induced in the body of the conductor, are called eddy currents. Because the resistance of the bulk conductor is usually low, eddy currents often have large magnitudes and heat up the conductor.

Eddy currents are always produced in a plane perpendicular to the direction of magnetic field. This current shows both heating and magnetic effects.

The magnitude of eddy current is given by

$$i = \frac{\text{induced emf}}{\text{resistance}} = \frac{e}{R}$$

According to Faraday's law,

$$e = -\frac{d\phi}{dt}, \quad i = -\frac{d\phi\,/dt}{R}$$

The direction of eddy currents can be given by Lenz's law or by Fleming's right hand rule.

However, their flow patterns resemble swirling eddies in water. That is why, they are called **eddy currents**. These were discovered by Foucault in the year 1895 and hence, they are also named as **Foucault current**, *e.g.*, when we move a metal plate out of a magnetic field, then the relative motion of the field and the conductor again induces a current in the conductor. The conducting electrons building up the induced current whirl around within the plate as, if they were caught in an eddy of water.

Effects of eddy current

Eddy currents are produced inside the iron cores of the rotating armatures of electric motors and dynamos and also in the cores of transformers, which experience flux changes, when they are in use. The currents causes unnecessary heating and wastage of power and the heat produced may even damage the insulation of coils.

Methods to Reduce eddy current

Eddy currents are minimised by using laminations of metal to make a metal core. The laminations are separated by an insulating material. The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths. This arrrangement reduces the strength of eddy currents.

Applications of eddy current

Eddy currents are useful in many ways. *Some of the important applications of eddy currents are given below*

(i) Electromagnetic damping

In order to bring the moving coil of a galvanometer immediately to rest, we make the use of electromagnetic damping which uses eddy currents to bring the coil to rest. When the coil oscillates, then the eddy currents generated in the core oppose the motion and bring the coil to rest.

(ii) Induction furnace

In an induction furnace, high temperature can be produced by using eddy currents. We generally use induction furnace in preparation of alloys by melting the constituents of metal.

A coil is wound over the metal which needs to be melted and through the coil, we pass high frequency alternating current. The eddy current generated in the metal produces high temperature to melt the metal.

(iii) Electric power meter

Old electric power meters (analog type) had a metallic disc. The disc rotates due to generation of eddy currents which are produced due to sinusoidally varying current in the coil.

(iv) Magnetic breaking in electronic trains

Some electric powered trains make use of strong electromagnets which are situated above the rails. These electromagnets are used to produce eddy current in the rails which oppose the motion of the train and thus stop it. In this case, as there is no mechanical linkage, the breaking effect is smooth.

Check point 6.4

1. Two coils X and Y are placed in a circuit such that when the current changes by 2 A in coil X. The magnetic flux changes by 0.4 Wb in Y. The value of mutual inductance of the coils is
(a) 0.2 H (b) 5 H (c) 0.8 H (d) 4 H

2. Two coils have a mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10$ A and $\omega = 100\pi$ rads^{-1}. The maximum value of emf in the second coil is (in volt)
(a) 2π (b) 5π (c) π (d) 4π

3. The mutual inductance between a primary and secondary circuits is 0.5 H. The resistance of the primary and the secondary circuits are 20 Ω and 5 Ω, respectively. To generate a current of 0.4 A in the secondary, current in the primary must be changed at the rate of
(a) 4.0 As^{-1} (b) 1.6 As^{-1} (c) 16.0 As^{-1} (d) 8.0 As^{-1}

4. The coefficient of mutual induction between two circuits is equal to the, emf produced in one circuit when the current in the second circuit is
(a) kept steady at 1 A
(b) cut-off at 1 A level
(c) changed at the rate of 1 As^{-1}
(d) changed from 1 As^{-1} to 2 As^{-1}

5. Two circuits have coefficient of mutual induction of 0.09 H. Average emf induced in the secondary by a change of current from 0 to 20 A in 0.006 s in the primary will be
(a) 120 V (b) 80 V (c) 200 V (d) 300 V

6. A current is varying at the rate of 3 A/s in a coil generates an emf of 8 mV in a nearby coil. The mutual inductance of the two coils is
(a) 2.66 ¬H (b) 2.66×10^{-3} mH
(c) 2.66 H (d) 0.266 H

7. A solenoid is placed inside another solenoid, the length of both being equal carrying same magnitude of current. The parameters like radius and number of turns are in the ratio 1: 2 for the two solenoids. The mutual inductance on each other would be
(a) $M_{12} = M_{21}$ (b) $M_{12} = 2M_{21}$
(c) $2M_{12} = M_{21}$ (d) $M_{12} = 4M_{21}$

8. Two coils of self-inductances 2 mH and 8 mH are placed, so close together that the effective flux in one coil is completely linked with the other. The mutual inductance between these coils is
(a) 4 mH (b) 16 mH
(c) 10 mH (d) 6 mH

9. Two coils of self-inductances L_1 and L_2 are placed closer to each other, so that total flux in one coil is completely linked with other. If M is mutual inductance between them, then
(a) $M = L_1 L_2$ (b) $M = L_1 / L_2$
(c) $M = \sqrt{L_1 L_2}$ (d) $M = (L_1 L_2)^2$

10. An ideal coil of 10 H is joined in series with a resistance of 5Ω and a battery of 5 V. 2 s after joining, the current flowing in ampere in the circuit will be
(a) e^{-1} (b) $(1-e^{-1})$ (c) $(1-e)$ (d) e

11. An L-R circuit has a cell of emf E, which is switched on at time $t = 0$. The current in the circuit after a long time will be
(a) zero (b) $\frac{E}{R}$ (c) $\frac{E}{L}$ (d) $\frac{E}{\sqrt{L^2 + R^2}}$

12. During current growth in an L-R circuit the time constant is the time in which the magnitude of current becomes
(a) I_0 (b) $I_0/2$ (c) $0.63\, I_0$ (d) $0.37\, I_0$

13. An L-R circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the connection?
(a) Current in the circuit
(b) Magnetic field energy
(c) Power delivered by the battery
(d) Emf induced in the inductor

14. Eddy currents are produced when
(a) a metal is kept in varying magnetic field
(b) a metal is kept in a steady magnetic field
(c) a circular coil is placed in a magnetic field
(d) through a circular coil current is passed

15. Which of the following is not an application of eddy currents?
(a) Induction furnace (b) Galvanometer damping
(c) Speedometer of automobiles (d) X-ray crystallography

Chapter exercises

A Taking it together

(Assorted questions of the chapter for advanced level practice)

1. If L, C and R represent the physical quantities inductance, capacitance and resistance respectively, only the following combination does not have the dimensions of frequency

(a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$

2. The self-inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as **[NCERT Exemplar]**

(a) Both l and A increase
(b) l decreases and A increases
(c) l increases and A decreases
(d) Both l and A decrease

3. In electromagnetic induction, the induced charge in a coil is independent of

(a) change in the flux
(b) time taken to change the flux
(c) resistance in the circuit
(d) None of the above

4. A coil and a bulb are connected in series with a DC source, a soft iron core is then inserted in the coil, then

(a) intensity of the bulb remains the same
(b) intensity of the bulb decreases
(c) intensity of the bulb increases
(d) the bulb ceases to glow

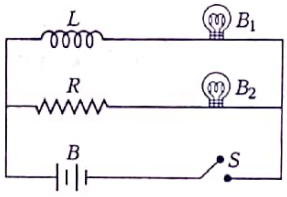
5. A magnet is brought towards a coil (i) speedly (ii) slowly, then the induced emf/induced charge will be respectively

(a) more in first case/more in first case
(b) more in first case/equal in both case
(c) less in first case/more in second case
(d) less in first case/equal in both case

6. In a coil of area 10 cm^2 and 10 turns, a magnetic field is directed perpendicular to the plane and is changing at the rate of 10^8 gauss/s. The resistance of the coil is 20 Ω. The current in the coil will be

(a) 5 A (b) 0.5 A (c) 0.05 A (d) 5×10^8 A

7. An inductor L, a resistance R and two identical bulbs, B_1 and B_2 are connected to a battery through a switch S as shown in the figure. Which of the following statements gives the correct description of the happenings when the switch S is closed?



(a) The bulb B_2 lights up earlier than B_1 and finally both the bulbs shine equally bright
(b) B_1 lights up earlier and finally both the bulbs acquire equal brightness
(c) B_2 lights up earlier and finally B_1 shines brighter than B_2
(d) B_1 and B_2 light up together with equal brightness all the time

8. Two identical circular loops of metal wires are lying on a table without touching each other. Loop A carries a current which increases with time. In response, the loop B

(a) remains stationary
(b) is attracted by loop A
(c) is repelled by loop A
(d) rotates about its centre of mass with CM fixed

9. A coil having 500 square loops each of side 10 cm is placed normal to a magnetic flux which increases at the rate of 1.0 Ts^{-1}. The induced emf in volts is

(a) 0.1 (b) 0.5
(c) 1 (d) 5

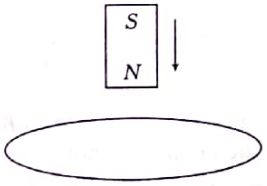
10. A coil having an area 2 m^2 is placed in a magnetic field which changes from 1 Wbm^{-2} to 4 Wbm^{-2} in an interval of 2 s. The emf induced in the coil will be

(a) 4 V (b) 3 V
(c) 1.5 V (d) 2 V

11. The circular loops of equal radii are placed coaxially at some separation. The first is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature.
During this period, the two loops
(a) attract each other
(b) repel each other
(c) do not exert any force on each other
(d) attract or repel each other depending on the sense of the current

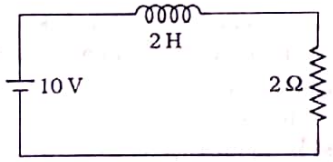
12. A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid.
If the current in the solenoid is varied, then the current induced in the loop is
(a) clockwise
(b) anti-clockwise
(c) zero
(d) clockwise or anti-clockwise depending on whether the resistance is increased or decreased

13. The north pole of a magnet is brought near a metallic ring as shown in the figure. The direction of induced current in the ring will be



(a) anti-clockwise
(b) clockwise
(c) first anti-clockwise and then clockwise
(d) first clockwise and then anti-clockwise

14. In the figure, magnetic energy stored in the coil is



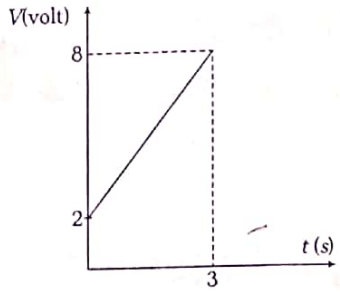
(a) zero
(b) infinite
(c) 25 J
(d) None of the above

15. An emf of 15 V is applied in a circuit coil containing 5 H inductance and 10 Ω resistance. The ratio of the currents at time $t = \infty$ and $t = 1$ s is

(a) $\frac{e^{1/2}}{e^{1/2}-1}$ (b) $\frac{e^2}{e^2-1}$

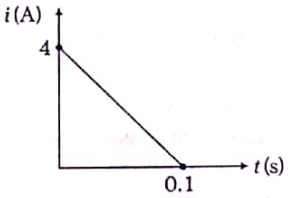
(c) $1-e^{-1}$ (d) e^{-1}

16. A circuit element is placed in a closed box. At time $t = 0$, a constant current generator supplying a current of 1 A is connected across the box. Potential difference across the box varies according to graph shown in the figure. The element in the box is



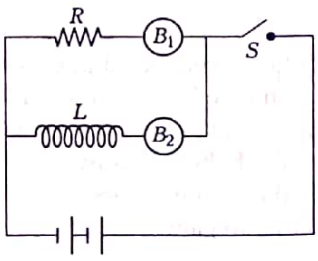
(a) a resistance of 2 Ω
(b) a battery of emf 6 V
(c) an inductance of 2 H
(d) a capacitance of 0.5 F

17. Some magnetic flux is changed from a coil of resistance 10 Ω. As a result an induced current is developed in it, which varies with time as shown in figure. The magnitude of change in flux through the coil in Wb is



(a) 2 (b) 4
(c) 6 (d) None of these

18. Figure shows two bulbs B_1 and B_2 resistor R and inductor L, when the switch S is turned off



(a) Both B_1 and B_2 die out promptly
(b) Both B_1 and B_2 die out with some delay
(c) B_2 dies out promptly but B_1 with some delay
(d) B_1 dies out promptly but B_2 with some delay

19. A coil of 40 Ω resistance, 100 turns and radius 6 mm is connected to ammeter of resistance 160 Ω. Coil is placed perpendicular to the magnetic field. When coil is taken out of the field, 32 μC charge flows through it. The intensity of magnetic field will be
(a) 6.55 T (b) 5.66 T
(c) 2.55 T (d) 0.566 T

20. A coil has an area of $0.05\ m^2$ and it has 800 turns. It is placed perpendicularly in a magnetic field of strength $4 \times 10^{-5}\ Wb\ m^{-2}$, and is rotated through 90° in 0.1 s. The average emf induced in the coil is

(a) 0.056 V (b) 0.046 V
(c) 0.026 V (d) 0.016 V

21. In a magnetic field of 0.05 T, area of a coil changes from $101\ cm^2$ to $100\ cm^2$ without changing the resistance which is 2 Ω. The amount of charge that flows during this period is

(a) 2.5×10^{-6} C
(b) 2×10^{-6} C
(c) 10^{-6} C
(d) 8×10^{-6} C

22. The resistance and inductance of series circuit are 5Ω and 20 H, respectively. At the instant of closing the switch, the current is increasing at the rate $4\ As^{-1}$. The supply voltage is

(a) 20 V (b) 80 V
(c) 120 V (d) 100 V

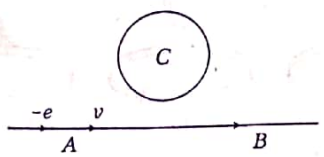
23. The number of turns in the coil of an AC generator is 5000 and the area of the coil is $0.25\ m^2$. The coil is rotated at the rate of 100 cycles/s in a magnetic field of $0.2\ Wm^{-2}$. The peak value of the emf generated is nearly

(a) 786 kV (b) 440 kV
(c) 220 kV (d) 157 kV

24. A wheel with ten metallic spokes each 0.50 m long is rotated with a speed of 120 rev/min in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 4.0 gauss, the induced emf between the axle and the rim of the wheel is equal to

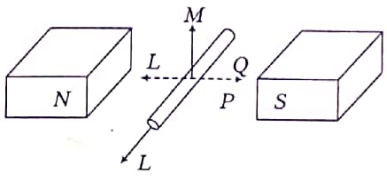
(a) 1.256×10^{-3} V
(b) 6.28×10^{-4} V
(c) 1.256×10^{-4} V
(d) 6.28×10^{-5} V

25. An electron moves along the line *AB* which lies in the same plane as a circular loop of conducting wire as shown in figure. What will be the direction of the current induced (if any) in the loop?



(a) No current will be induced
(b) The current will be clockwise
(c) The current will be anti-clockwise
(d) The current will change direction as the electron passes through

26. An electric potential difference will be induced between ends of the conductor shown in the diagram, when the conductor moves in the direction

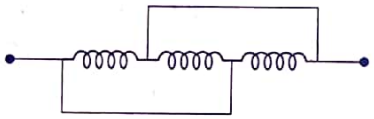


(a) *P* (b) *Q* (c) *L* (d) *M*

27. A long horizontal metallic rod with length along the east-west direction is falling under gravity. The potential difference between its two ends will

(a) be zero (b) be constant
(c) increase with time (d) decrease with time

28. All the three inductors have inductance 3.0 H. The equivalent inductance of the circuit is



(a) 1 H (b) 2 H (c) 3 H (d) 9 H

29. Two inductances connected in parallel are equivalent to a single inductance of 1.5 H and when connected in series are equivalent to a single inductance of 8 H. The difference in their inductance is

(a) 3 H (b) 7.5 H (c) 2 H (d) 4 H

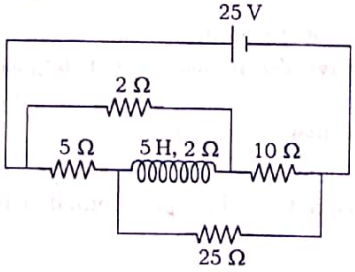
30. An inductance *L* and a resistance *R* are first connected to a battery. After some time the battery is disconnected but *L* and *R* remain connected in a closed circuit. Then the current reduces to 37% of its initial value in

(a) *RL* second (b) $\frac{R}{L}$ second
(c) $\frac{L}{R}$ second (d) $\frac{1}{LR}$ second

31. The time constant of an inductance coil is 2.0×10^{-3} s. When a 90 Ω resistance is joined in series, then the time constant becomes 0.5×10^{-3} s. The inductance and resistance of the coil are

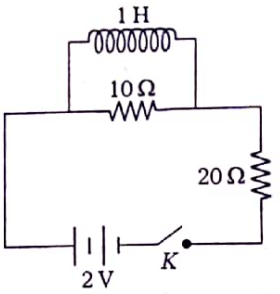
(a) 30 mH, 30 Ω (b) 30 mH, 60 Ω
(c) 60 mH, 30 Ω (d) 60 mH, 60 Ω

32. In the circuit shown, what is the energy stored in the coil at steady state?



(a) 21.3 J (b) 42.6 J (c) Zero (d) 213 J

33. In the following figure, what is the final value of current in the 10 ohm resistor when the plug of key K is inserted?



(a) (3/10) A
(b) (3/20) A
(c) (3/11) A
(d) Zero

34. A square loop of side l, resistance R is placed in a uniform magnetic field B acting normally to the plane of the loop. If we attempt to pull it out of the field with a constant velocity v, then the power needed is

(a) $BRlv$ (b) $\frac{B^2l^2v^2}{R}$

(c) $\frac{Bl^2v^2}{R}$ (d) $\frac{Bvl}{R}$

35. A square of side L metres lies in the xy-plane in a region, where the magnetic field is given by $\mathbf{B} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T, where B_0 is constant. The magnitude of flux passing through the square is **[NCERT Exemplar]**

(a) $2B_0L^2$ Wb
(b) $3B_0L^2$ Wb
(c) $4B_0L^2$ Wb
(d) $\sqrt{29}B_0L^2$ Wb

36. A conducting loop of area 5.0 cm^2 is placed in a magnetic field which varies sinusoidally with time as $B = 0.2 \sin 300\, t$. The normal to the coil makes an angle of 60° with the field. The emf induced at $t = (\pi / 900)$s

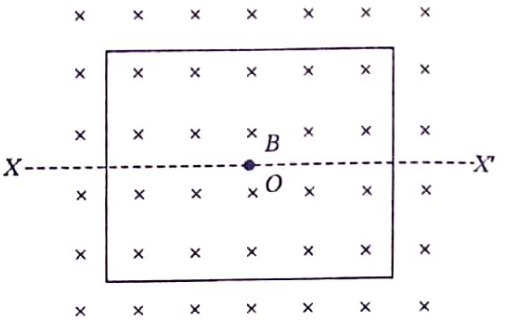
(a) 7.5×10^{-3} V (b) zero

(c) 15×10^{-3} V (d) 20×10^{-3} V

37. An infinitely long cylinder is kept parallel to a uniform magnetic field **B** directed along positive Z-axis. The direction of induced current as seen from the Z-axis will be

(a) clockwise of the positive z-axis
(b) anti positive Z-axis clockwise of the positive z-axis
(c) zero
(d) along the magnetic field

38. A rectangular coil is placed in a region having a uniform magnetic field B perpendicular to the plane of the coil. An emf will not be induced in the coil, if the



(a) magnetic field is increased uniformly
(b) magnetic field is switched off
(c) coil is rotated about an axis XX'
(d) coil is rotated about an axis perpendicular to the plane of the coil and passing through its centre O

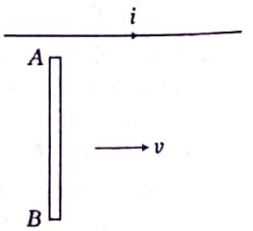
39. If a coil of 40 turns and area 4.0 cm^2 is suddenly removed from a magnetic field, then it is observed that a charge of 2.0×10^{-4} C flows into the coil. If the resistance of the coil is 80Ω, then the magnetic flux density in Wb/m^2 is

(a) 0.5 (b) 1.0
(c) 1.5 (d) 2.0

40. A coil of wire of a certain radius has 600 turns and a self-inductance of 108 mH. The self-inductance of a 2nd similar coil of 500 turns will be

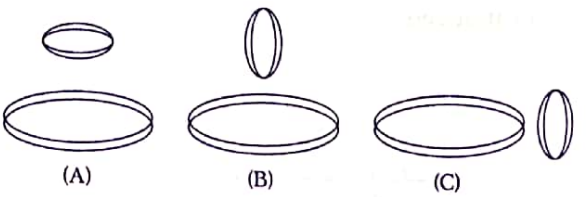
(a) 74 mH (b) 75 mH
(c) 76 mH (d) 77 mH

41. The current carrying wire and the rod AB are in the same plane. The rod moves parallel to the wire with a velocity v. Which one of the following statements is true about induced emf in the rod?



(a) End A will be at lower potential with respect to B
(b) A and B will be at the same potential
(c) There will be no induced emf in the rod
(d) Potential at A will be higher than that at B

42. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be

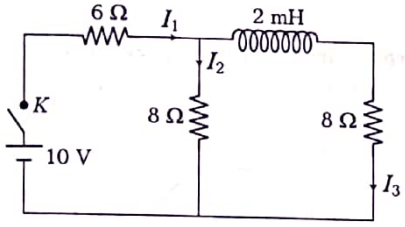


(a) maximum in situation (A)
(b) maximum in situation (B)
(c) maximum in situation (C)
(d) the same in all situations

43. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in
(a) 0.15 s (b) 0.3 s (c) 0.05 s (d) 0.1 s

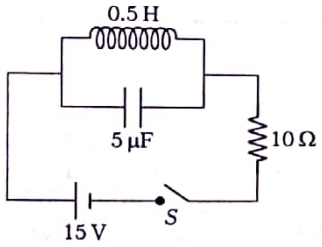
44. An inductor of 2 H and a resistance of 10 Ω are connected in series with a battery of 5 V. The initial rate of change in current is
(a) 0.5 As^{-1} (b) 2.0 As^{-1}
(c) 2.5 As^{-1} (d) 0.25 As^{-1}

45. In the circuit shown in the figure, what is the value of I_1 just after pressing the key K?



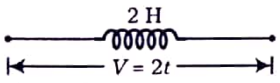
(a) 5/7 A (b) 5/11 A
(c) 1 A (d) None of these

46. For in the circuit shown in figure, current through the battery at $t = 0$ and $t = \infty$ is



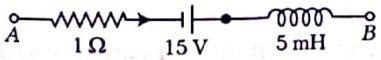
(a) 1.5 A, 1.5 A (b) 0, 0
(c) 1, 1.5 A (d) 1.5 A, 0

47. A variable voltage $V = 2t$ is applied across an inductor of inductance $L = 2$ H as shown in figure. Then,



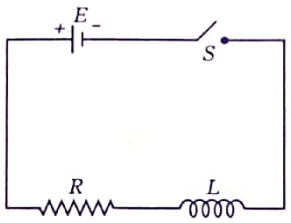
(a) current *versus* time graph is a parabola
(b) energy stored in magnetic field at $t = 2$ s is 4 J
(c) potential energy at time $t = 1$ s in magnetic field is increasing at a rate of 1 J s^{-1}
(d) energy stored in magnetic field is zero all the time.

48. The network shown in the figure is a part of a complete circuit. If at a certain instant the current i is 5 A and decreasing at the rate of 10^3 As^{-1}, then $V_B - V_A$ is

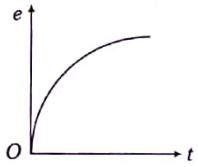


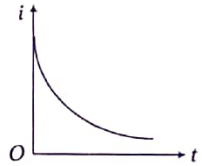
(a) 5 V (b) 10 V (c) 15 V (d) 20 V

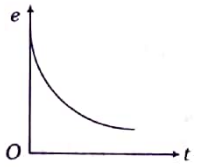
49. Switch S of the circuit shown in figure is closed at $t = 0$.

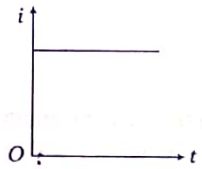


If emf in L is e and i is the current flowing through the circuit at time t, which of the following graphs is correct?

(a)


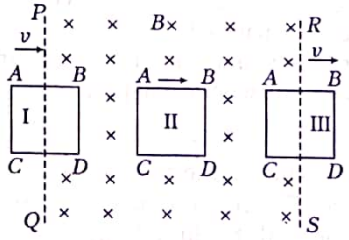
(b)


(c)


(d)


50. Coefficient of coupling between two coils of self-inductances L_1 and L_2 is unity. It means
(a) 50% flux of L_1 is linked with L_2
(b) 100% flux of L_1 is linked with L_2
(c) $\sqrt{L_1}$ time of flux of L_1 is linked with L_2
(d) None of the above

51. A square loop $ABCD$ of edge 'a' moves to the right with a velocity v, parallel to AB. There is a uniform magnetic field of magnitude B, directed into the paper, in the region between PQ and RS only. I, II and III are three positions of the loop.

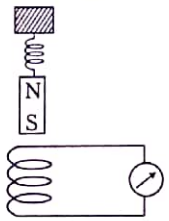


(i) The emf induced in the loop has magnitude Bav in all three positions.
(ii) Induced emf is zero in position II.
(iii) The induced emf is anti-clockwise in position I
(iv) The induced emf is clockwise in position III.
(a) (i), (iii)
(b) (ii), (iii), (iv)
(c) (i), (ii)
(d) (iii), (iv)

52. A rectangular, a square, a circular and an elliptical loop, all in the XY-plane, are moving out of a uniform magnetic field with a constant velocity $\mathbf{v} = u\hat{\mathbf{i}}$. The magnetic field is directed along the negative Z-axis direction. The induced emf during the passage of these loops, out of the field region, will not remain constant for

(a) the rectangular, circular and elliptical loops
(b) the circular and the elliptical loops
(c) only the elliptical loop
(d) any of the four loops

53. A magnet N-S is suspended from a spring and while it oscillates, the magnet moves in and out of the coil C. The coil is connected to a galvanometer G.



Then, as the magnet oscillates,

(a) G shows deflection to the left and right with constant amplitude
(b) G shows deflection on one side
(c) G shows no deflection
(d) G shows deflection to the left and right but the amplitude steadily decreases

54. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating, then it is very difficult to stop. But, if an aluminium plate is placed near to the coil, then it stops. This is due to

(a) development of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material
(d) electromagnetic induction in the aluminium plate give rise to electromagnetic damping

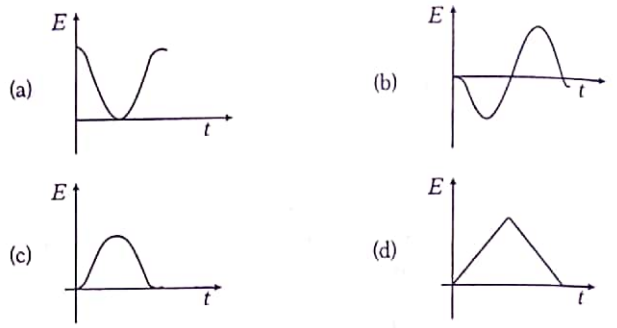
55. Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. If, then $R_1 >> R_2$, then the mutual inductance M between them will be directly proportional to

(a) R_1 / R_2 (b) R_2 / R_1
(c) R_1^2 / R_2 (d) R_2^2 / R_1

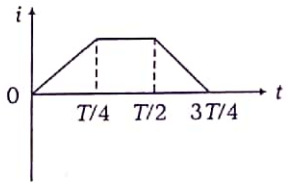
56. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, then the electrical power dissipated would be

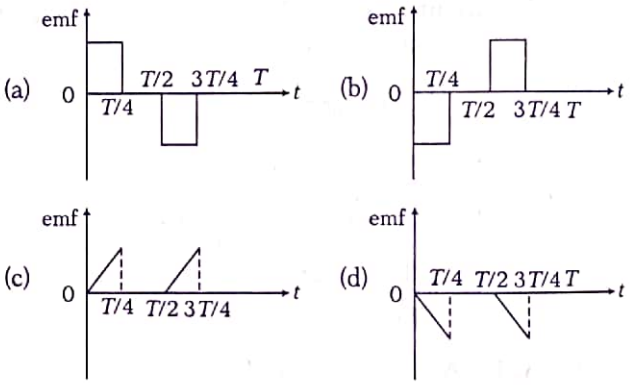
(a) halved (b) the same
(c) doubled (d) quadrupled

57. The variation of induced emf (E) with time (t) in a coil, if a short bar magnet is moved along its axis with a constant velocity is best represented as

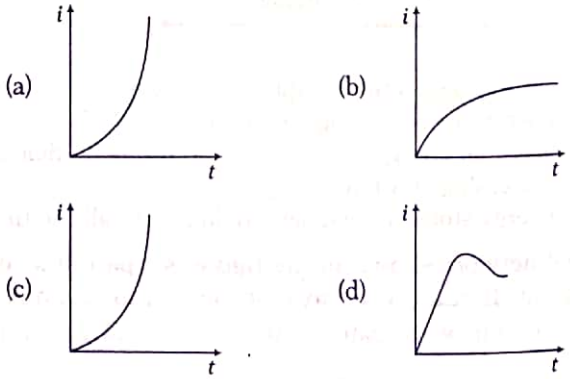


58. The current i in a coil varies with time as shown in the figure. The variation of induced emf with time would be



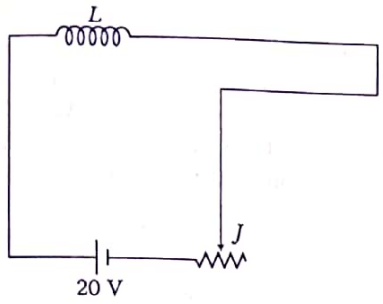


59. When a battery is connected across a series combination of self-inductance L and resistance R, then the variation in the current i with time t is best represented by



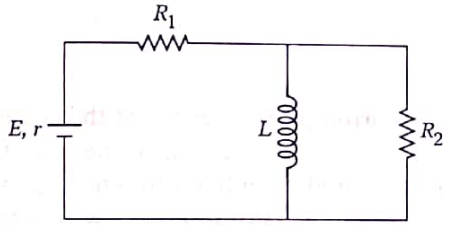
60. In the circuit shown in the figure, the jockey J is being pulled towards right, so that the resistance in the circuit is increasing. Its a value at some instant is 5 Ω.

The current in the circuit at this instant will be



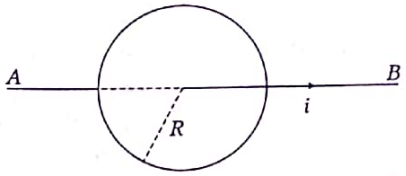
(a) 4 A
(b) less than 4 A
(c) more than 4 A
(d) may be less than or more than 4 A depending on the value of L

61. The value of time constant for the given circuit is



(a) $\frac{L}{R_1 + r + R_2}$ (b) $\frac{L}{(R_1 + r)}$
(c) $\frac{L(R_1 + R_2 + r)}{(R_1 + r)R_2}$ (d) None of these

62. An infinitely long conductor AB lies along the axis of a circular loop of radius R. If the current in the conductor AB varies at the rate of x ampere/second, then the induced emf in the loop is



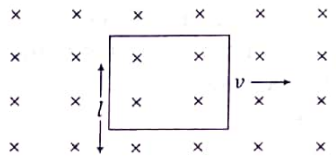
(a) $\frac{\mu_0 xR}{2}$ (b) $\frac{\mu_0 xR}{4}$ (c) $\frac{\mu_0 \pi xR}{2}$ (d) zero

63. A physicist works in a laboratory where the magnetic field is 2 T. She wears a necklace of enclosing area 0.01 m^2 in such a way that the plane of the necklace is normal to the field and is having a resistance $R = 0.01\ \Omega$. Because of power failure, the field decays to 1 T in time 10^{-3} s. Then, what is the total heat produced in her necklace?
(a) 10 J (b) 20 J (c) 30 J (d) 40 J

64. A wheel with ten metallic spokes each 0.50 m long is rotated with a speed of 120 rev min^{-1} in a plane normal to the earth's magnetic field at the plane. If the magnitude of the field is 0.4 G, the induced emf between the axle and the rim of the wheel is equal to
(a) 1.256×10^{-3} V (b) 6.28×10^{-4} V
(c) 1.256×10^{-4} V (d) 6.28×10^{-5} V

65. An aeroplane in which the distance between the tips of the wings is 50 m is flying horizontally from east to west with the speed of 360 kmh^{-1} over a place where the vertical component of the earth's magnetic field is 2.0×10^{-4} Wbm^{-2}. The potential difference between the tips of the wings would be
(a) 0.1 V (b) 0.01 V (c) 0.2 V (d) 1.0 V

66. A conducting square loop of side l and resistance R is moving out of the plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop as shown in figure.



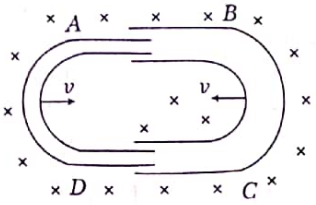
The current induced in the loop is
(a) $\frac{Blv}{R}$ clockwise (b) $\frac{Blv}{R}$ anti-clockwise
(c) $\frac{2Blv}{R}$ anti-clockwise (d) zero

67. Two rails of a railway track, insulated from each other and the ground, are connected to a millivoltmetre. What is the reading of the millivoltmetre when a train travels at a speed of 20 ms^{-1} along the track? Given that the vertical component of earth's magnetic field is 0.2×10^{-4} Wbm^{-2} and the rails are separated by 1 m
(a) 4 mV (b) 0.4 mV
(c) 80 mV (d) 10 mV

68. A pair of coils of turns n_1 and n_2 are kept close together. Current passing through the first is reduced at the rate r, and emf 3 mV is developed across the other coil. If the second coil carries current which is then reduced at the rate $2r$, then the emf produced across the first coil will be
(a) $\frac{6n_1}{n_2}$ mV (b) $\frac{6n_2}{n_1}$ mV (c) 6 mV (d) 3/2 mV

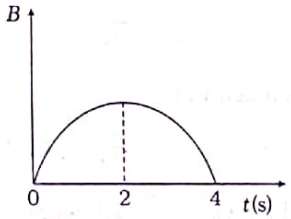
69. A loop made of straight edges has six corners at A (0, 0, 0), B (L, 0, 0), C (L, L, 0), D (0, L, 0), E (0, L, L) and F (0, 0, L). A magnetic field $B = B_0(\hat{i} + \hat{k})$ T is present in the region. The flux passing through the loop $ABCDEFA$ (in that order) is **[NCERT Exemplar]**
(a) B_0L^2 Wb (b) $2B_0L^2$ Wb (c) $\sqrt{2}B_0L^2$ Wb (d) $4B_0L^2$ Wb

70. One conducting U-tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed u, then the emf induced in the circuit in terms of B, l and v, where l is the width of each tube, will be



(a) zero (b) $2Blv$
(c) Blv (d) $-Blv$

71. In a closed loop, which has some inductance but negligible resistance, uniform but time varying magnetic field is applied directed into the plane of the loop. Variation of field with time is shown. Initially, current in the loop was zero. Then,

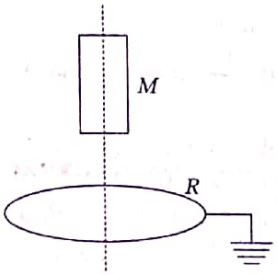


(a) emf induced in the loop is zero at $t = 2$ s
(b) current in the loop will be maximum at $t = 2$ s
(c) direction of emf in the loop will change at $t = 2$ s
(d) None of the above

72. A magnetic field given by $B(t) = 0.2t - 0.05t^2$ tesla is directed perpendicular to the plane of a circular coil containing 25 turns of radius 1.8 cm and whose total resistance is 1.5 Ω. The power dissipation at 3 s is approximately

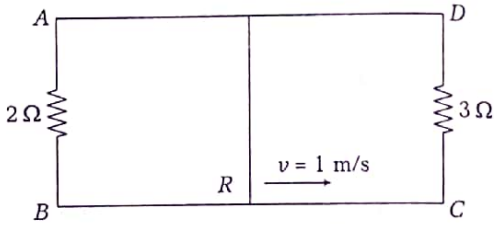
(a) 1.37 μW (b) 7 μW (c) zero (d) 4 μW

73. A small magnet M is allowed to fall through a fixed horizontal conducting ring R. Let g be the acceleration due to gravity. The acceleration of M, a will be



(a) $< g$ when it is above R and moving towards R
(b) $> g$ when it is above R and moving towards R
(c) $< g$ when it is below R and moving away from R
(d) $> g$ when it is below R and moving away from R

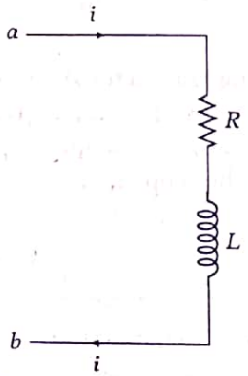
74. A rectangular loop with a sliding connector of length 10 cm is situated in uniform magnetic field perpendicular to plane of loop. The magnetic induction is 0.1 T and resistance of connector (R) is 1Ω. The sides AB and CD have resistances 2Ω and 3 Ω, respectively. Find the current in the connector during its motion with constant velocity 1 ms^{-1}.



(a) $\frac{1}{242}$ A (b) $\frac{1}{220}$ A
(c) $\frac{1}{55}$ A (d) $\frac{1}{440}$ A

75. When the current in the portion of the circuit shown in the figure is 2 A and increasing at the rate of 1 As^{-1}, then the measured potential difference $V_{ab} = 8$ V. However, when the current is 2A and decreasing at the rate of 1As^{-1}, then the measured potential difference $V_{ab} = 4$ V.

The values of R and L are

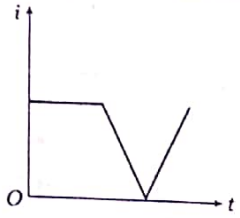


(a) 3 Ω and 2 H respectively
(b) 3 Ω and 3 H respectively
(c) 2 Ω and 1 H respectively
(d) 3 Ω and 1 H respectively

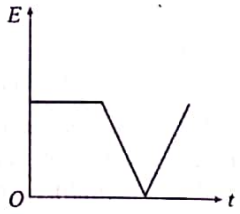
76. Two different coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in second coil is also increased at the same constant rate. At certain instant of time, the power given to the coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2, respectively. Then, choose the wrong option.

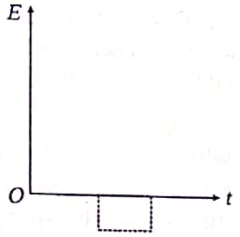
(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$ (c) $\frac{W_1}{W_2} = \frac{1}{4}$ (d) $\frac{V_2}{V_1} = \frac{1}{4}$

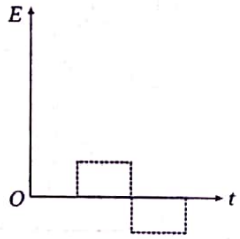
77. The current i in an induction coil varies with time t according to the graph shown in figure.

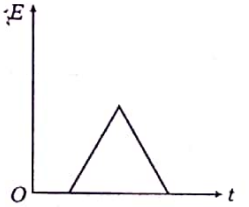


Which of the following graphs shows the induced emf (E) in the coil with time?

(a)


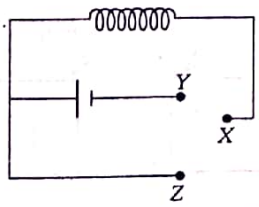
(b)


(c)


(d)


78. A coil of inductance $L = 50\,\mu H$ and resistance $= 0.5\,\Omega$ is connected to a battery of emf $= 5$ V.
A resistance of $10\,\Omega$ is connected parallel to the coil. Now, at same instant the connection of the battery is switched off. Then, the amount of heat generated in the coil after switching off the battery is
(a) 1.25 mJ (b) 2.5 mJ (c) 0.65 mJ (d) 0.12 mJ

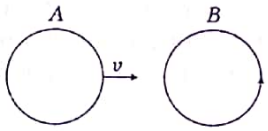
79. In the circuit shown, the coil has inductance and resistance. When X is joined to Y, the time constant is τ during growth of current. When the steady state is reached, then heat is produced in the coil at a rate P. If X is now joined to Z, then choose the correct statement.



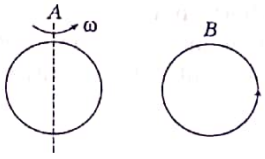
(a) the total heat produced in the coil is $P\tau$
(b) the total heat produced in the coil is $\frac{1}{2}P\tau$
(c) the total heat produced in the coil is $2P\tau$
(d) the data given is not sufficient to reach a conclusion

80. A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then, **[NCERT Exemplar]**
(a) a direct current flows in the ammeter A
(b) no current flows through the ammeter A
(c) an alternating sinusoidal current flows through the ammeter A with a time period $T = \frac{2\pi}{\omega}$
(d) a time varying non-sinusoidal current flows through the ammeter A

81. There are two coils A and B as shown in figure. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter-clockwise. B is kept stationary when A moves. We can infer that **[NCERT Exemplar]**
(a) there is a constant current in the clockwise direction in A
(b) there is a varying current in A
(c) there is no current in A
(d) there is a constant current in the counter-clockwise direction in A



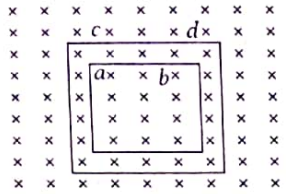
82. Same as above problem except, the coil A is made to rotate about a vertical axis (figure). No current flows in B, if A is at rest. The current in coil A,when the current in B (at $t = 0$) is counter-clockwise and the coil A is as shown at this instant, $t = 0$, is **[NCERT Exemplar]**
(a) constant current clockwise
(b) varying current clockwise
(c) varying current counter-clockwise
(d) constant current counter-clockwise



83. A rectangular loop of sides 10 cm and 5 cm with a cut is stationary between the pole pieces of an electromagnet.

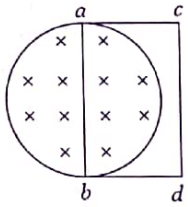
The magnetic field of the magnet is normal to the loop. The current feeding the electromagnet is reduced, so that the field decreased from its initial value of 0.2 T at the rate of $0.02\,\Omega$. If the cut is joined and the loop has a resistance of 2.0Ω, then the power dissipated by the loop as heat is
(a) 5 nW
(b) 4 nW
(c) 3 nW
(d) 2 nW

84. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments *ab* and *cd*. Then,



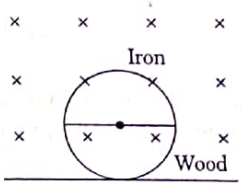
(a) $I_1 > I_2$
(b) $I_1 < I_2$
(c) I_1 is in the direction *ba* and I_2 is in the direction *cd*
(d) I_1 is in the direction *ab* and I_2 is in the direction *dc*

85. A uniform magnetic field B exists in a cylindrical region of radius 10 cm as shown in figure. A uniform wire of length 80 cm and resistance 4.0 Ω is bent into a square frame and is placed with one side along a diameter of the cylindrical region. If the magnetic field increases at a constant rate of 0.010 T/s, then the current induced in the frame



(a) 3.9×10^{-5} A (b) 4.0×10^{5} A
(c) 4.1×10^{-5} A (d) 3.9×10^{-4} A

86. Figure shows a circular wheel of radius 10.0 cm whose upper half, is made of iron and the lower half of wood as shown in figure. The two junctions are joined by an iron rod. A uniform magnetic field B of magnitude 2.0×10^{-4} T exists in the space above the central line as suggested by the figure. The wheel is set into pure rolling on the horizontal surface. If it takes 2.0 s for the iron part to come down and the wooden part to go up, then the average emf induced during this period is

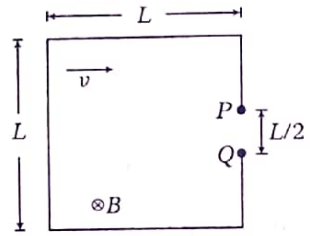


(a) 1.57×10^{-6} V (b) 1.5×10^{5} V
(c) 15.7×10^{-6} V (d) 1.55×10^{-6} V

87. A wire of length 10 cm translates in a direction making an angle of 60° with its length. The plane of motion is perpendicular to a uniform magnetic field of 1.0 T that exists in the space. The emf induced between the ends of the rod, if the speed of translation is 20 cm/s is

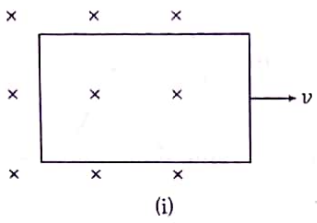
(a) 1.7×10^{-3} V (b) 17×10^{-3} V
(c) 0.17×10^{-3} V (d) 1.7×10^{-4} V

88. The loop shown moves with a velocity v in a uniform magnetic field of magnitude B, directed into the paper. The potential difference between P and Q is e



(i) $e = BLv / 2$
(ii) $e = BLv$
(iii) P is positive with respect to Q
(iv) Q is positive with respect to P
(a) (ii), (iii) (b) (i), (iv) (c) (i), (iii) (d) (ii), (iv)

89. A conducting loop being pulled out of magnetic field with a speed v. Which of the following plots may represent the power delivered by the pulling agent as a function of the speed v?

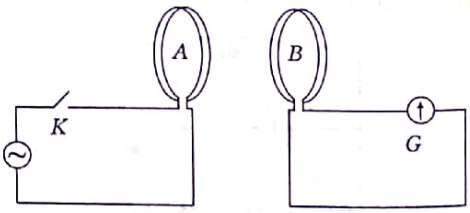


(a) *a* (b) *b* (c) *c* (d) *d*

90. A square loop of wire with side length 10 cm is placed at angle of 45° with a magnetic field that changes uniformly from 0.1 T to zero in 0.7s. The induced current in the loop (its resistance is 1Ω) is

(a) 1.0 mA (b) 2.5 mA (c) 3.5 mA (d) 4.0 mA

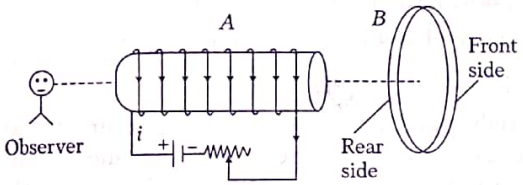
91. The diagram below shows two coils A and B placed parallel to each other at a very small distance. Coil A is connected to an AC supply. G is a very sensitive galvanometer. When the key is closed



(a) constant deflection will be observed in the galvanometer for 50 Hz supply
(b) visible small variations will be observed in the galvanometer for 50 Hz input
(c) oscillations in the galvanometer may be observed when the input AC voltage has a frequency of 1 to 2 Hz
(d) No variation will be observed in the galvanometer even when the input AC voltage is 1 or 2 Hz

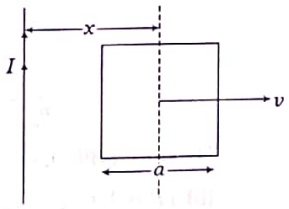
92. A magnet is dropped down an infinitely long vertical copper tube,
(a) the magnet moves with continuously increasing velocity and ultimately acquires a constant terminal velocity
(b) the magnet moves with continuously decreasing velocity and ultimately comes to rest
(c) the magnet moves with continuously increasing velocity but constant acceleration
(d) the magnet moves with continuously increasing velocity and acceleration

93. An aluminium ring B faces an electromagnet A. The current I through A can be altered



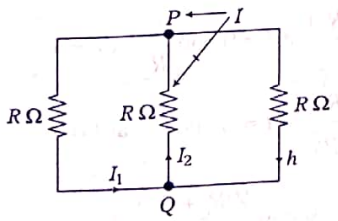
(a) whether I increases or decreases, B will not experience any force
(b) if I decreases, A will repel B
(c) if I increases, A will attract B
(d) if I increases, A will repel B

94. A conducting square frame of side a and a long straight wire carrying current I are located in the same plane as shown in the figure. The frame moves to the right with a constant velocity v. The emf induced in the frame will be proportional to



(a) $\frac{1}{(2x-a)^2}$ (b) $\frac{1}{(2x+a)^2}$
(c) $\frac{1}{(2x-a)(2x+a)}$ (d) $1/x^2$

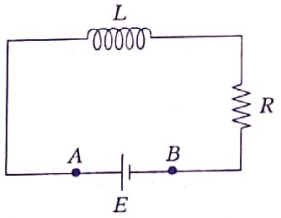
95. A rectangular loop has a sliding connector PQ of length l and resistance $R\,\Omega$ and it is moving with a speed v as shown. The set up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(a) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$ (b) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$
(c) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$ (d) $I_1 = I_2 = I = \frac{Blv}{R}$

96. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 A and 4 A are the currents flowing in each coil, respectively. The magnetic induction in Wb / m^2 at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb / A-m)
(a) 12×10^{-5} (b) 10^{-5}
(c) 5×10^{-5} (d) 7×10^{-5}

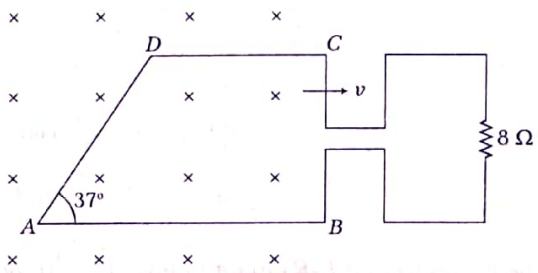
97. An inductor ($L = 100$ mH), a resistor ($R = 100\Omega$) and a battery ($E = 100$V) are initially connected in series as shown in the figure. After a long time, the battery is disconnected after short-circuiting the points A and B. The current in the circuit 1 ms after the short-circuit is



(a) eA (b) 0.1 A
(c) 1 A (d) $1/e$ A

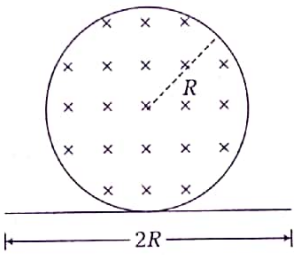
98. Three solenoid coils of same dimension, same number of turns and same number of layers of windings are taken. Coil 1 with inductance L_1 was wound using a wire of resistance 11 Ω / m, coil 2 with inductance L_2 was wound using the similar wire but the direction of winding was reversed in each layer, coil 3 with inductance L_3 was wound using a superconducting wire. The self-inductance of the coils L_1, L_2 and L_3 are
(a) $L_1 = L_2 = L_3$
(b) $L_1 = L_2, L_3 = 0$
(c) $L_1 = L_3, L_2 = 0$
(d) $L_1 > L_2 > L_3$

99. The loop $ABCD$ is moving with velocity v towards right. The magnetic field is 4 T. The loop is connected to a resistance of 8 Ω. If steady current of 2 A flows in the loop, then value of v, if loop has resistance of 4 Ω, is (Given, $AB = 30$ cm, $AD = 30$ cm, $\sin 37° = \frac{3}{5}$)



(a) $\frac{50}{3}$ ms^{-1} (b) 20 ms^{-1}
(c) 10 ms^{-1} (d) $\frac{100}{3}$ ms^{-1}

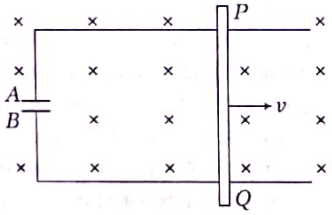
100. A uniform but time varying magnetic field is present in a circular region of radius R. The magnetic field is perpendicular and into the plane of the loop and the magnitude of field is increasing at a constant rate α. There is a straight conducting rod of length $2R$ placed as shown in figure.



The magnitude of induced emf across the rod is

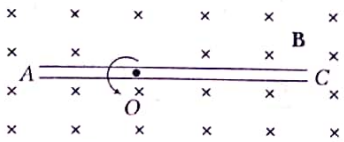
(a) $\pi R^2\alpha$ (b) $\frac{\pi R^2\alpha}{2}$ (c) $\frac{R^2\alpha}{\sqrt{2}}$ (d) $\frac{\pi R^2\alpha}{4}$

101. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 2\ \text{ms}^{-1}$ in a uniform magnetic field $B = 4.0$ T directed into the paper. A capacitor of capacity $C = 10\ \mu$F is connected as shown in figure. Then,



(a) $q_A = +80\ \mu$C and $q_B = -80\ \mu$C
(b) $q_A = -80\ \mu$C and $q_B = +80\ \mu$C
(c) $q_A = 0 = q_B$
(d) charge stored in the capacitor increases exponentially with time

102. A conducting rod AC of length $4l$ is rotated about a point O in a uniform mangnetic field **B** directed into the paper. $AO = l$ and $OC = 3l$. Then,



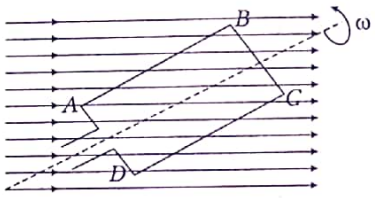
(a) $V_A - V_O = \frac{B\omega l^2}{2}$ (b) $V_O - V_C = \frac{7}{2}B\omega l^2$

(c) $V_A - V_C = 4B\omega l^2$ (d) $V_C - V_O = \frac{9}{2}B\omega l^2$

103. The current in an L-R circuit builds upto $\frac{3}{4}$th of its steady state value in 4 s. The time constant of this circuit is

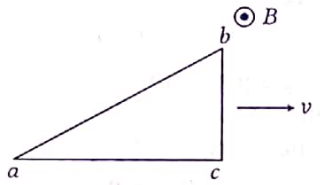
(a) $\frac{1}{\ln 2}$ s (b) $\frac{2}{\ln 2}$ s (c) $\frac{3}{\ln 2}$ s (d) $\frac{4}{\ln 2}$ s

104. A rectangular coil $ABCD$ which is rotated at a constant angular velocity about an horizontal as shown in the figure. The axis of rotation of the coil as well as the magnetic field B are horizontal. Maximum current will flow in the circuit when the plane of the coil is



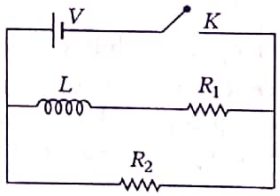
(a) inclined at 30° to the magnetic field
(b) perpendicular to the magnetic field
(c) inclined at 45° to the magnetic field
(d) parallel to the magnetic field

105. A right-angled triangle abc, made from a metallic wire, moves at a uniform speed v in its plane as shown in figure. A uniform magnetic field B exists in the perpendicular direction. The emf induced in the loop abc, in the segment bc, in the segment ac and in the segment ab are



(a) zero; $vB(bc)$,+ve at c, zero, $vB(bc)$,+ve at a
(b) $vB(bc)$,+ve at c, zero; zero; $vB(bc)$,+ve at a
(c) zero; zero; $vB(bc)$,+ve at c, $vB(bc)$,+ve at a
(d) $vB(bc)$,+ve at c, $vB(bc)$,+ve at a; zero; zero

106. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is



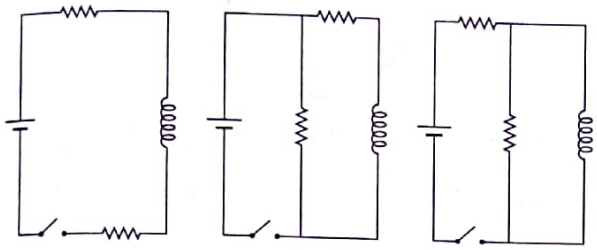
(a) $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{V(R_1+R_2)}{\sqrt{R_1^2+R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = \infty$

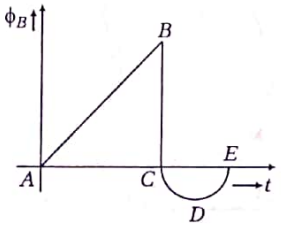
(d) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{\sqrt{R_1^2+R_2^2}}$ at $t = \infty$

107. The figure shows three circuits with identical batteries, inductors and resistors. Rank the circuits, in the decreasing order, according to the current through the battery (i) just after the switch is closed and (ii) a long time later



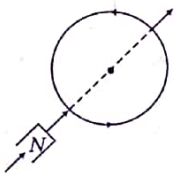
(a) (i) $i_2 > i_3 > i_1$, $(i_1 = 0)$ (ii) $i_2 > i_3 > i_1$
(b) (i) $i_2 < i_3 < i_1$, $(i_1 \neq 0)$ (ii) $i_2 > i_3 > i_1$
(c) (i) $i_2 = i_3 = i_1$, $(i_1 = 0)$ (ii) $i_2 < i_3 < i_1$
(d) (i) $i_2 = i_3 > i_1$, $(i_1 \neq 0)$ (ii) $i_2 > i_3 > i_1$

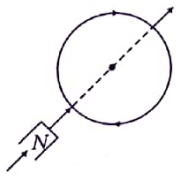
108. The graph shows the variation in magnetic flux $\phi(t)$ with time through a coil. Which of the statements given below is not correct?

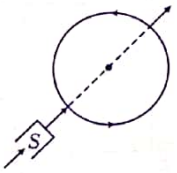


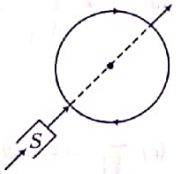
(a) There is a change in the direction as well as magnitude of the induced emf between B and D
(b) The magnitude of the induced emf is maximum between B and C
(c) There is a change in the direction as well as magnitude of induced emf between A and C
(d) The induced emf is not zero at B

109. Which of the following figure correctly depicts the Lenz's law. The arrows show the movement of the labelled pole of a bar magnet into a closed circular loop and the arrows on the circle show the direction of the induced current

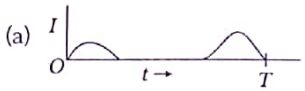
(a) 

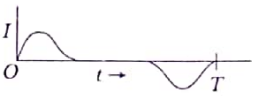
(b) 

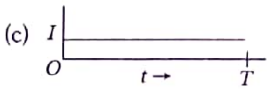
(c) 

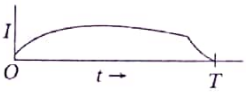
(d) 

110. A metallic ring is dropped down, keeping its plane perpendicular to a constant and horizontal magnetic field. The ring enters the region of magnetic field at $t = 0$ and completely emerges out at $t = T$ sec. The current in the ring varies as

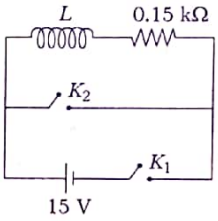
(a) 

(b) 

(c) 

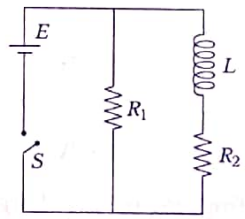
(d) 

111. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15\Omega$) are connected in series to a battery of 15 V emf in a circuit shown below. The key K_1 has been kept closed for a long time. Then, at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ m/s the current in the circuit will be $(e^5 \cong 150)$



(a) 100 mA (b) 67 mA (c) 6.7 mA (d) 0.67 mA

112. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is

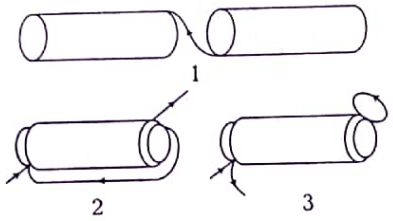


(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V (c) $6\left(1 - e^{\frac{-1}{0.2}}\right)$ V (d) $12e^{-5t}$ V

113. There are two solenoids of same length and inductance L but their diameters differ to the extent that one can just fit into the other. They are connected in three different ways in series.

(1) They are connected in series but separated by large distance.
(2) They are connected in series with one inside the other and senses of the turns coinciding.
(3) Both are connected in series with one inside the other with senses of the turns opposite,

as depicted in figures 1, 2 and 3, respectively. The total inductance of the solenoids in each of the case 1, 2 and 3 are respectively.



(a) $0, 4L_0, 2L_0$
(b) $4L_0, 2L_0, 0$
(c) $2L_0, 0, 4L_0$
(d) $2L_0, 4L_0, 0$

114. A rectangular loop of length l and breadth b is placed at distance of x from infinitely long wire carrying current i such that the direction of current is parallel to breadth. If the loop moves away from the current wire in a direction perpendicular to it with a velocity v, then magnitude of the emf in the loop is (μ_0 = permeability of free space)

(a) $\frac{\mu_0 iv}{2\pi x}\left(\frac{l+b}{b}\right)$ (b) $\frac{\mu_0 i^2 v}{4\pi^2 x}\log\left(\frac{b}{l}\right)$

(c) $\frac{\mu_0 ilbv}{2\pi x(l+x)}$ (d) $\frac{\mu_0 ilbv}{2\pi}\log\left(\frac{x+l}{x}\right)$

115. A circular coil of one turn of radius 5.0 cm is rotated about a diameter with a constant angular speed of 80 revolutions per minute. A uniform magnetic field $B = 0.01$ T exists in a direction perpendicular to the axis of rotation, the maximum emf induced, the average emf induced in the coil over a long period and the average of the squares of emf induced over a long period is

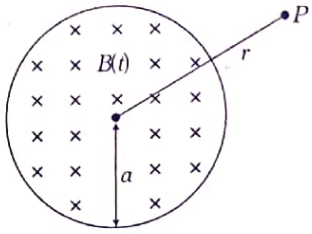
(a) 6.4×10^{-4} V, zero, 2.2×10^{-7} V^2
(b) 6.6×10^{-4} V, zero, 2.0×10^{-7} V^2
(c) 6.8×10^{-4} V, zero 2.5×10^{-7} V^2
(d) 6.4×10^{-4} V, zero 2.0×10^{-6} V^2

116. A non-conducting ring having charge q uniformly distributed over its circumference is placed on a rough horizontal surface. A vertical time varying magnetic field $B = 4t^2$ is switched on at time $t = 0$. Mass of the ring is m and radius is R.

The ring starts rotating after 2 s, the coefficient of friction between the ring and the table is

(a) $\frac{4qmR}{g}$ (b) $\frac{2qmR}{g}$ (c) $\frac{8qR}{mg}$ (d) $\frac{qR}{2mg}$

117. A uniform but time-varying magnetic field $B(t)$ exists in a cylindrical region of radius a and is directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region

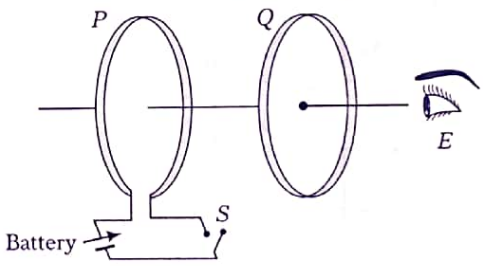


(a) is zero (b) decreases as $\frac{1}{r}$

(c) increases as r (d) decreases as $\frac{1}{r^2}$

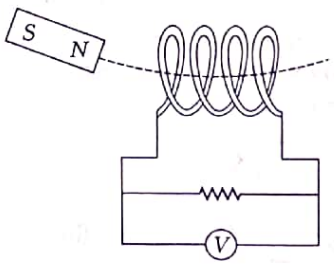
118. As shown in the figure, P and Q are two co-axial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q_1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q_2} flows in Q.

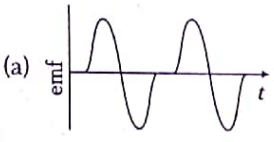
Then, the directions of I_{Q_1} and I_{Q_2} (as seen by E) are

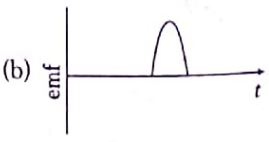


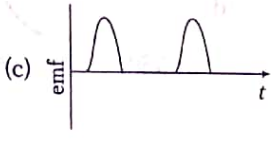
(a) respectively clockwise and anti-clockwise
(b) both clockwise
(c) both anti-clockwise
(d) respectively anti-clockwise and clockwise

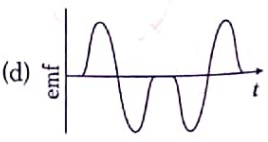
119. A magnet is made to oscillate with a particular frequency, passing through a coil as shown in figure. The time variation of the magnitude of emf generated across the coil during one cycle.



(a) 

(b) 

(c) 

(d) 

B Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-17) *These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose anyone of the following five responses.*

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If Both Assertion and Reason are false.

1. **Assertion** Magnetic flux and the electric flux have the same units.
Reason Flux passing through a surface gives an idea about the field lines crossing that surface.

2. **Assertion** In the phenomenon of mutual induction, self-induction of each of the coils persists.
Reason Self-induction arises when strength of current in same coil changes. In mutual induction, current is changing in both the individual coils.

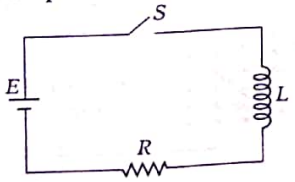
3. **Assertion** When two magnets are brought closer to each other, then they will always repel each other.
Reason According to Lenz's law induced effects always opposes the cause.

4. **Assertion** Mutual inductance of two coils depends on the distance between the coils and their orientation.
Reason It does not depend on the magnetic material filled between the coils.

5. **Assertion** Coefficient of self-induction of an inductor depends upon the rate of change of current passing through it.
Reason From, $e = -L\dfrac{di}{dt}$
We can see that, $L = \dfrac{-e}{(di/dt)}$ or $L \propto \dfrac{1}{(di/dt)}$

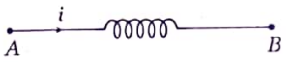
6. **Assertion** If a magnet is brought closer to a current carrying loop along its axis, then current always decreases in the loop.
Reason Magnet is repelled by the loop.

7. **Assertion** In the figure, just after closing the switch the potential drop across inductor is maximum.



Reason The rate of change of current just after closing the switch is maximum.

8. **Assertion** If current shown in the figure is increasing, then
$V_A > V_B$



Reason If current passing through an inductor is constant, then both ends of the inductor are at same potential.

9. **Assertion** If two inductors are in parallel, then current in them distributes in inverse ratio of their inductance.
Reason In parallel, potential difference remains constant.

10. **Assertion** An induced emf of 2 V is developed in a circular loop, if current in the loop is changed at a rate of 4 As^{-1}. If 4 A of current is passed through this loop, then flux linked with this coil will be 2 Wb.
Reason Flux linked with the coil is
$$|\phi| = \left|\frac{e}{di/dt}\right| i$$

11. **Assertion** If a straight wire is moved in a magnetic field, no emf will be induced across its two ends as the circuit is not closed.
Reason Since, the circuit is not closed, induced current will be zero.

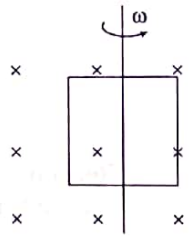
12. **Assertion** If a loop is placed in a non-uniform (with respect to position) magnetic field, then induced emf is produced in the loop.
Reason In a non-uniform magnetic field, magnetic flux passing through the loop will change. Therefore, induced emf is produced.

13. **Assertion** If current passing through a circular loop is doubled, then magnetic flux linked with the circular loop will also become two times.
Reason No flux will link through the coil by its own current.

14. **Assertion** Two concentric conducting rings of different radii are placed in space. The mutual inductance of both the rings is maximum, if the rings are coplanar.
Reason For two co-axial conducting rings of different radii, the magnitude of magnetic flux in one ring due to current in other ring is maximum when both rings are coplanar.

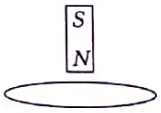
15. Assertion The induced emf and current will be same in two identical loops of copper and aluminium when rotated with same speed in the same magnetic field.
Reason Induced emf is proportional to rate of change of magnetic field while induced current depends on resistance of wire.

16. Assertion A conducting loop is rotated in a uniform magnetic field with constant angular velocity ω as shown in figure. At time $t = 0$, plane of the loop is perpendicular to the magnetic field. Induced emf produced in the loop is maximum when plane of loop is parallel to magnetic field.



Reason When plane of loop is parallel to magnetic field, then magnetic flux passing through the loop is zero.

17. Assertion A magnet is dropped along the axis of a circular conducting loop as shown in figure. Then, acceleration of magnet is always less than g.



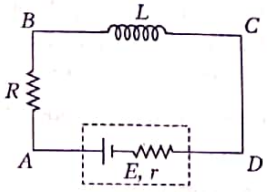
Reason When magnet is above the loop, then it will repel the magnet and when it is below the loop, then it will attract the magnet.

Match the columns

1. Match the following columns.

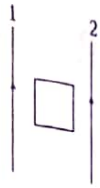
	Column I		Column II
A.	Tesla	p.	$[ML^2A^{-2}T^{-2}]$
B.	Weber	q.	$[MLA^{-2}T^{-1}]$
C.	Weber m^{-2}	r.	$[MA^{-1}T^{-2}]$
D.	Henry	s.	None

2. In the circuit shown in figure, $E = 10\ V$, $r = 1\ \Omega$, $R = 4\ \Omega$ and $L = 5\ H$. The circuit is closed at time $t = 0$. Then, match the following two columns.



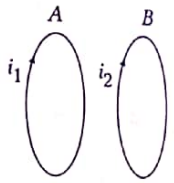
	Column I		Column II
A.	V_{AD} at $t = 0$	p.	4 V
B.	V_{BC} at $t = 0$	q.	8 V
C.	V_{AB} at $t = \infty$	r.	0 V
D.	V_{AD} at $t = \infty$	s.	10 V

3. A square loop is symmetrically placed between two infinitely long current carrying wires in the same direction. Magnitude of currents in both the wires are same. Now, match the following two columns.



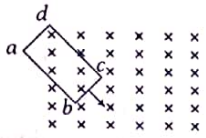
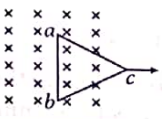
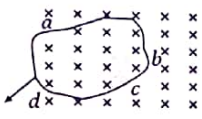
	Column I		Column II
A.	Loop is moved towards right	p.	Induced current in the loop is clockwise.
B.	Loop is moved towards left	q.	Induced current in the loop is anti-clockwise.
C.	Wire-1 is moved towards left	r.	Induced current in the loop is zero.
D.	Wire-2 is moved towards right	s.	Induced current in the loop is non-zero.

4. Two co-axial identical circular current carrying loops are shown in figure, currents in them are in the same directions. Now, match the following two columns.



	Column I		Column II
A.	Current i_1 is increased	p.	Loops will attract each other
B.	Current i_2 is decreased	q.	Loops will repel each other
C.	Loop-1 is moved towards loop-2	r.	Current i_1 will increase
D.	Loop-2 is moved away from loop-1	s.	Current i_2 will increase

5. Match the items of Column I with those of Column II.

	Column I (Planar loops of different shapes)		Column II (Direction of induced current)
A.		p.	*b a c b*
B.		q.	*c d a b c*
C.		r.	*b c d a b*

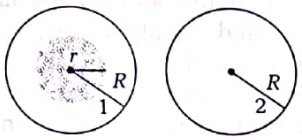
C. Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

1. A long solenoid has 1000 turns. When a current of 4 A flows through it, then the magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is **[NEET 2016]**

(a) 3 H (b) 2 H
(c) 1 H (d) 4 H

2. A uniform magnetic field is restricted within a region of radius r. The magnetic field changes with time at a rate $\frac{dB}{dt}$. Loop 1 of radius $R > r$ encloses the region r and loop 2 of radius R is outside the region of magnetic field as shown in the figure. Then, the emf generated is **[NEET 2016]**



(a) Zero in loop 1 and zero in loop 2
(b) $-\frac{dB}{dt}\pi r^2$ in loop 1 and $-\frac{dB}{dt}\pi r^2$ in loop 2
(c) $-\frac{dB}{dt}\pi R^2$ in loop 1 and zero in loop 2
(d) $-\frac{dB}{dt}\pi r^2$ in loop 1 and zero in loop 2

3. The self-inductance of a coil having 500 turns is 50 mH. The magnetic flux through the cross-sectional area of the coil while current through it is 8 mA is found to be **[AIIMS 2015]**

(a) 4×10^{-4} Wb (b) 0.04 Wb
(c) 4 μWb (d) 40 mWb

4. The phase difference between the flux linked with a coil rotating in a uniform magnetic field and induced emf produced in it is **[AIIMS 2015]**

(a) $\pi/2$ (b) $\pi/3$
(c) $-\pi/6$ (d) π

5. A rectangular copper coil is placed in a uniform magnetic field of induction 40 mT with its plane perpendicular to the field. The area of the coil is shrinking at a constant rate of 0.5 $m^2 s^{-1}$. The emf induced in the coil is **[EAMCET 2015]**

(a) 10 mV (b) 20 mV (c) 80 mV (d) 40 mV

6. Changing magnetic fields can set up current loops in nearby metal bodies and the currents are called as **[Kerala CEE 2015]**

(a) eddy currents (b) flux currents
(c) alternating currents (d) leaking currents
(e) wattless currents

7. A rod of 10 cm length is moving perpendicular to uniform magnetic field of intensity 5×10^{-4} Wbm^{-2}. If the acceleration of the rod is 5 ms^{-1}, then the rate of increase of induced emf is **[Guj. CET 2015]**

(a) $25\times 10^{-4} Vs^{-1}$ (b) $2.5\times 10^{-4} Vs^{-1}$
(c) $20\times 10^{-4} Vs^{-1}$ (d) $20\times 10^{-4} Vs^{-1}$

8. The identical loops of copper and aluminium are moving with the same speed in a magnetic field. Which of the following statements is true? **[CG PMT 2015]**

(a) Induced emf and induced current are same in both loops
(b) Induced emf remains same, induced current changes in both loops
(c) Induced emf changes but induced current remains same in both loops
(d) Induced emf will be more in aluminium loop

9. A straight conductor 0.1 m long moves in a uniform magnetic field 0. 1 T. The velocity of the conductor is 15 ms^{-1} and is directed perpendicular to the field. The emf induced between the two ends of the conductor is **[WB JEE 2015]**

(a) 0.10 V (b) 0.15 V (c) 1.50 V (d) 15 V

10. The initial rate of increase of current, when a battery of emf 6 V is connected in series with an inductance of 2 H and resistance 12 Ω, is **[CG PMT 2015]**

(a) 0.5 As^{-1} (b) 1 As^{-1} (c) 3 As^{-1} (d) 3.5 As^{-1}

11. The current flows from A to B as shown in the figure. What is the direction of current in circle? **[UK PMT 2014]**

(a) Clockwise (b) Anti-clockwise
(c) Straight line (d) None of these

12. A very small circular loop of radius a is initially (at $t=0$) coplanar and concentric with a much larger fixed circular loop of radius b. A constant current I flows in the larger loop. The smaller loop is rotated with a constant angular speed ω about the common diameter. The emf induced in the smaller loop as a function of time t is **[WB JEE 2014]**

(a) $\frac{\pi a^2 \mu_0 I}{2b}\omega\cos\omega t$ (b) $\frac{\pi a^2 \mu_0 I}{2b}\omega\sin\omega^2 t^2$
(c) $\frac{\pi a^2 \mu_0 I}{2b}\omega\sin\omega t$ (d) $\frac{\pi a^2 \mu_0 I}{2b}\omega\sin^2\omega t$

13. A straight conductor of length 0.4 m is moved with a speed of 7 ms^{-1} perpendicular to a magnetic field of intensity 0.9 Wbm^{-2}. The induced emf across the conductor is **[UK PMT 2014]**

(a) 5.04 V (b) 1.26 V (c) 2.52 V (d) 25.2 V

14. The current in a self-inductance $L = 40$ mH is to be increased uniformly from 1A to 11 A in 4 ms. The emf induced in the inductor during the process is **[UK PMT 2014]**

(a) 100 V (b) 0.4 V (c) 40 V (d) 440 V

15. The induced emf in a coil of 10 H inductance in which current varies from 9 A to 4 A in 0.2 s is **[J & K CET 2013]**

(a) 200 V (b) 250 V
(c) 300 V (d) 350 V

16. A conductor of length 5 cm is moved parallel to itself with a speed of 2 m/s, perpendicular to a uniform magnetic field of 10^{-3} Wb/m^3. The induced emf generated is **[J & K CET 2013]**

(a) 2×10^{-3}V (b) 1×10^{-3}V
(c) 1×10^{-4}V (d) 2×10^{-4}V

17. Two identical circular coils A and B are kept on a horizontal tube side by side without touching each other. If the current in the coil A increases with time, in response, the coil B **[Karnataka CET 2013]**

(a) is attracted by A (b) remains stationary
(c) is repelled (d) rotates

18. A rectangular coil of 100 turns and size 0.1 m× 0.05 m is placed perpendicular to a magnetic field of 0.1 T. If the field drops to 0.05 T in 0.05 s, the magnitude of the emf induced in the coil is **[Karnataka CET 2013]**

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{0.6}$ (d) $\sqrt{6}$

19. Electromagnetic induction is not used in **[Kerala CET 2013]**

(a) speedometer (b) transformer
(c) AC generator (d) induction furnace
(e) room heater

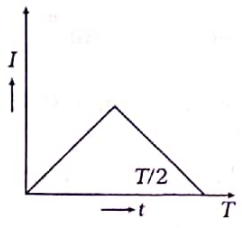
20. Two coils have the mutual inductance of 0.05 H. The current changes in the first coil as $I = I_0 \sin \omega t$, where $I_0 = 1$ A and $\omega = 100\pi$ rad/s. The maximum emf induced in secondary coil is **[UP CPMT 2013]**

(a) 2.5 V (b) 10 V (c) 6πV (d) 5πV

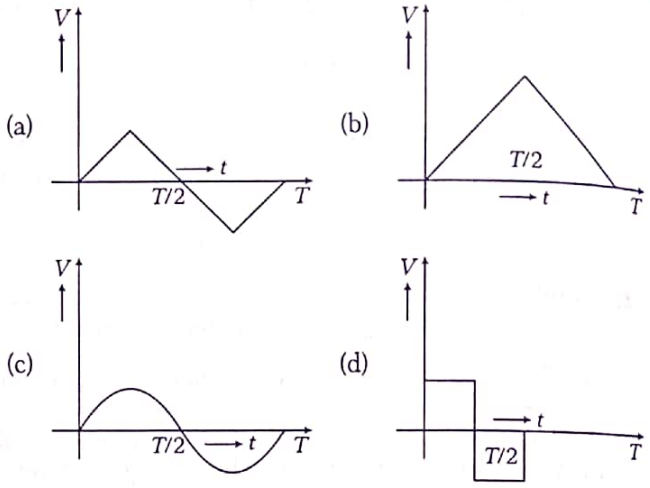
21. The magnetic flux linked with a circuit of resistance 100 Ω increases from 10 to 60 Wb. The amount of induced charge that flows in the circuit is (in coulomb) **[UP CPMT 2012]**

(a) 0.5 (b) 5 (c) 50 (d) 100

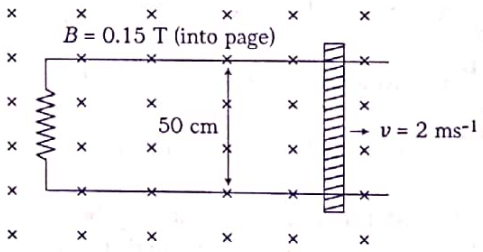
22. The current (I) in the inductance is varying with time according to the plot shown in figure.



Which one of the following is the correct variation of voltage with time in the coil? **[CBSE AIPMT 2012]**



23. As shown in the figure, a metal rod makes contact with a partial circuit and completes the circuit. The circuit area is perpendicular to a magnetic field with $B = 0.15$ T. If the resistance of the total circuit is 3 Ω, the force needed to move the rod as indicated with a constant speed of 2 ms^{-1} will be equal to **[AMU 2012]**



(a) 3.75×10^{-3} N
(b) 2.75×10^{-3} N
(c) 6.57×10^{-4} N
(d) 4.36×10^{-4} N

24. If emf induced in a coil is 2 V by changing the current in it from 8 A to 6 A in 2×10^{-3} s, then the coefficient of self-induction is **[AFMC 2012]**

(a) 2×10^{-3} H (b) 10^{-3} H
(c) 0.5×10^{-3} H (d) 4×10^{-3} H

25. A wire of length 1m is moving at a speed of 2 ms^{-1} perpendicular to its length in a homogeneous magnetic field of 0.5T. If the ends of the wire are joined to a circuit of resistance 6 Ω, then the rate at which work is being done to keep the wire moving at constant speed is **[JCECE 2012]**

(a) 1W (b) $\frac{1}{3}$W
(c) $\frac{1}{6}$W (d) $\frac{1}{12}$W

Answers

Check point 6.1

1. (c)	**2.** (b)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (b)
11. (b)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (a)					

Check point 6.2

1. (b)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (c)
11. (c)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (a)					

Check point 6.3

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (d)	**9.** (d)	**10.** (a)
11. (a)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (c)					

Check point 6.4

1. (a)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (b)
11. (b)	**12.** (c)	**13.** (a)	**14.** (a)	**15.** (d)					

A. Taking it together

1. (d)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (d)	**10.** (b)
11. (d)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (d)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (d)
21. (a)	**22.** (b)	**23.** (d)	**24.** (d)	**25.** (d)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (d)	**30.** (c)
31. (c)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (c)	**36.** (a)	**37.** (c)	**38.** (d)	**39.** (b)	**40.** (b)
41. (d)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (a)	**46.** (a)	**47.** (a,b,c)	**48.** (c)	**49.** (c)	**50.** (b)
51. (b)	**52.** (b)	**53.** (d)	**54.** (d)	**55.** (d)	**56.** (b)	**57.** (b)	**58.** (b)	**59.** (b)	**60.** (c)
61. (c)	**62.** (d)	**63.** (a)	**64.** (d)	**65.** (d)	**66.** (a)	**67.** (b)	**68.** (c)	**69.** (b)	**70.** (b)
71. (a,c)	**72.** (d)	**73.** (a,c)	**74.** (b)	**75.** (a)	**76.** (b)	**77.** (c)	**78.** (d)	**79.** (b)	**80.** (b)
81. (d)	**82.** (a)	**83.** (a)	**84.** (d)	**85.** (a)	**86.** (a)	**87.** (b)	**88.** (c)	**89.** (b)	**90.** (a)
91. (c)	**92.** (a)	**93.** (d)	**94.** (c)	**95.** (c)	**96.** (c)	**97.** (d)	**98.** (b)	**99.** (d)	**100.** (d)
101. (a)	**102.** (c)	**103.** (b)	**104.** (d)	**105.** (a)	**106.** (c)	**107.** (a)	**108.** (d)	**109.** (a)	**110.** (b)
111. (d)	**112.** (d)	**113.** (d)	**114.** (c)	**115.** (b)	**116.** (c)	**117.** (b)	**118.** (d)	**119.** (a)	

B. Medical entrance special format questions

Assertion and reason

1. (d)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (a)
11. (d)	**12.** (e)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (b)	**17.** (a)			

Match the columns

1. (A → r; B → s; C → r; D → p)
2. (A → s; B → s; C → q; D → q)
3. (A → p, s; B → q, s; C → p, s; D → q, s)
4. (A → q; B → p; C → q, r; D → p,s)
5. (A → r; B → p; C → q)

C. Medical entrances' gallery

1. (c)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (c)
11. (d)	**12.** (c)	**13.** (c)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (*)	**19.** (e)	**20.** (d)
21. (a)	**22.** (d)	**23.** (a)	**24.** (a)	**25.** (c)					

*None of the option is correct.

Hints and explanations

Check point 6.1

1. (*c*) Magnetic flux linked with a vector area **A** in a uniform magnetic field **B** is given by $\phi = \mathbf{B} \cdot \mathbf{A}$

2. (*b*) Magnetic flux is a scalar quantity. Its SI unit is weber.

3. (*d*) Dimensional formula of magnetic flux $= [ML^2 T^{-2} A^{-1}]$

4. (*d*) Magnitude of induced emf, $e = -\frac{d\phi}{dt}$

$$= -\frac{d}{dt}(3t^2 + 4t + 9) = -(6t + 4)$$

$$= -[6 \times (2) + 4] = -16$$

$\therefore \quad |e| = 16$ V

5. (*a*) As, $|e| = \frac{\Delta\phi}{\Delta t} = \frac{4B_0 A_0 - B_0 A_0}{t} = \frac{3B_0 A_0}{t}$

6. (*d*) Induced emf, $e = \left|\frac{d\phi}{dt}\right| = |12t - 5|$

At $t = 0.25$ s $\Rightarrow e = 2$ V

$\therefore$ Induced current, $i = \frac{e}{R} = 0.2$ A

8. (*b*) Magnetic flux, $\phi = (5t^2 - 4t + 1)$ Wb

$\therefore \quad \frac{d\phi}{dt} = 10t - 4$ Wbs^{-1}

The induced emf, $e = \frac{-d\phi}{dt} = -(10t - 4)$

At, $t = 0.2$ s, $e = -(10 \times 0.2 - 4) = 2$ V

The induced current, $I = \frac{e}{R} = \frac{2\text{ V}}{10\ \Omega} = 0.2$ A

9. (*b*) As, charge, $|\Delta q| = \frac{\Delta\phi}{R} = \frac{(2)\left(\frac{\pi}{4}\right)(0.2)^2}{0.01} = 6.28$ C

10. (*b*) As, charge, $\Delta Q = \frac{\Delta\phi}{R} = \frac{(10 - 2)}{2} = 4$ C

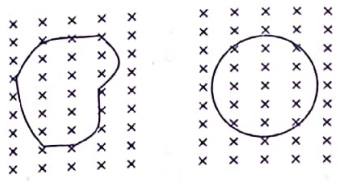
12. (*c*) According to Lenz's law, the induced current will be in such a direction, so that it opposes the change due to which it is produced.

13. (*c*) Lenz's law is based upon law of conservation of energy.

14. (*d*) According to Lenz's law, it will be anti-clockwise.

15. (*a*) Due to change in the shape of the loop, the magnetic flux linked with the loop increases. Hence, current is induced in the loop in such a direction that it opposes the increases in flux.

Therefore, induced current flows in the anti-clockwise direction.



Check point 6.2

1. (*b*) The potential difference between two ends of conducting rods will be Bvl.

2. (*c*) When a wire of length l moves with velocity v, perpendicular to a magnetic field B, then the induced emf is produced. The magnitude of induced emf is given by $|\varepsilon| = Blv$.

Given $l = 50$ cm $= 0.5$ m, $v = 300$ m/min $= 5$ m/s

$$|\varepsilon| = 2\text{V or } B = \frac{|\varepsilon|}{lv} = \frac{2}{0.5 \times 5} = 0.8\text{ T}$$

3. (*b*) Induced emf $= Blv = 0.3 \times 10^{-4} \times 10 \times 5$

$$= 1.5 \times 10^{-3}\text{V} = 1.5\text{ mV}$$

4. (*d*) Induced emf $= B \times v \times l$

$$= 5.0 \times 10^{-5} \times 1.50 \times 2 = 10.0 \times 10^{-5} \times 1.5$$

$$= 1.5 \times 10^{-4} = 0.15\text{ mV}$$

5. (*b*) When a conductor lying along the magnetic north-south, then moves eastwards, it will cut vertical component of B_0. So, induced emf,

$$e = vB_v\, l = v(B_0 \sin\delta l) = B_0 lv \sin\delta$$

6. (*b*) Induced emf, $e = B \cdot \frac{dS}{dt} = 2\pi rB \cdot \frac{dr}{dt}$

where, $\frac{dr}{dt}$ is the rate at which radius of loop is shrinking,

7. (*c*) Two circular loops carrying current in the same direction will attract each other. If they are now separated, then current will increase in both in accordance with Lenz's law.

8. (*b*) Induced emf, $e_0 = \omega NBS = (2\pi\nu)NB(\pi r^2) = 2 \times \pi^2 \nu NBr^2$

$$= 2 \times (3.14)^2 \times \frac{1800}{60} \times 4000 \times 0.5 \times 10^{-4} \times (7 \times 10^{-2})^2 = 0.57\text{ V}$$

10. (*c*) Here, $l = 0.1$m, $v = 1$ms^{-1}

$$I = 50\text{ A and } B = 1.25\text{ mT} = 1.25 \times 10^{-3}\text{ T}$$

The induced emf is, $\varepsilon = Blv$.

The mechanical power is,

$$P = \varepsilon I = BlvI = 1.25 \times 10^{-3} \times 0.1 \times 1 \times 50$$

$$= 6.25 \times 10^{-3}\text{W} = 6.25\text{ mW}$$

11. (c) Induced emf, when rod rotates in a vertical plane perpendicular to magnetic field is given as

$$e = \frac{1}{2}Bl^2\omega = \frac{1}{2} \times 0.3 \times 2^2 \times 100 = 60 \text{ V}$$

13. (c) $E = E_m \cos 45°$,

where, E_m is the maximum value of induced emf.

$$\therefore \quad E = \frac{E_m}{\sqrt{2}}$$

Check point 6.3

1. (b) $U = \frac{1}{2}Li^2$, $[L] = \left[\frac{U}{i^2}\right]$

and $$L = \frac{e}{di/dt}$$

$$L = \phi_B / i$$

$$\Rightarrow \quad [L] = [\phi/i][L] = [edt/di] = \frac{\text{volt-second}}{\text{ampere}}$$

2. (a) We have, $N\phi_B = Li$

Self-inductance, $L = \frac{N\phi_B}{i} = \frac{500 \times 4 \times 10^{-3}}{2} = 1\text{H}$

3. (d) Self-inductance of coil,

$$|e| = L\frac{di}{dt}$$

$$\therefore \quad 10 = L \times \frac{10}{1}$$

$\therefore$ Self-inductance, $L = 1$ H

4. (b) Self-inductance, $L = \left|\frac{e}{di/dt}\right| = \frac{8}{(2/0.5)} = 2$ H

5. (a) Given, $\frac{di}{dt} = 2$ A/s and $L = 5$ H

$\therefore$ The emf developing across the coil,

$$e = L\frac{di}{dt} = 5 \times 2 = 10 \text{ V}$$

6. (c) We have, $\frac{\Delta i}{\Delta t} = \frac{10}{2} = 5$ V

emf, $e = L\frac{\Delta i}{\Delta t}$

$= 0.5 \times 5 = 2.5$V

7. (d) The self-inductance of a long solenoid is given by

$$L = \frac{\mu_r \mu_0 n^2 A}{l}$$

It is clear that the self-inductance of a long solenoid does not depend upon the current flowing through it.

8. (d) Here, $L = 50 \times 10^{-3}$ H

$$\frac{dI}{dt} = \frac{(1-0)}{0.1} = 10$$

emf, $e = \frac{L \cdot dI}{dt} = 50 \times 10^{-3} \times 10$

$= 50 \times 10^{-2} = 0.5$V

9. (d) Self-inductance of coil is directly proportional to square of number of turns in the coil, *i.e.*,

$$L \propto N^2$$

$$\therefore \quad \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$$

$$\therefore \quad \frac{L}{L_2} = \frac{N^2}{(4N)^2}$$

$$\therefore \quad L_2 = 16L$$

10. (a) $L \propto N^2$

So, when number of turns is doubled L' will become 4 times of L, *i.e.*, $L' = 4L$ or 4 times.

11. (a) Self-inductance of solenoid is given by

$$\frac{\mu_0 N^2 S}{L}$$

12. (d) Induced emf, $e = \frac{Li}{dt} = \frac{\mu_0 N^2 S}{l}\frac{di}{dt}$

$$= \frac{4\pi \times 10^{-7} \times (2000)^2 \times 1.2 \times 10^{-3}}{0.30} \times \left(\frac{4}{0.25}\right) = 321 \times 10^{-2} \text{ V}$$

13. (b) Energy stored in the coil,

$$E = \frac{1}{2}Li^2 = \frac{1}{2} \times 50 \times 10^{-3} \times 4 = 0.1 \text{ J}$$

14. (a) $E = \frac{1}{2}Li^2 = \frac{1}{2}(100 \times 10^{-3})(10)^2 = 5$ J

15. (c) Equivalent inductance, $\frac{1}{L_{eq}} = \frac{1}{L} + \frac{1}{L}$

$\therefore$ The total inductance, $L_{eq} = \frac{L}{2}$

Check point 6.4

1. (a) We have, $M = \frac{\Delta\phi_2}{\Delta i_1}$

$$M = \frac{0.4}{2}$$

$\therefore$ Mutual inductance, $M = 0.2$ H

2. (b) Value of induced emf in coil,

$$e = M\frac{di}{dt}$$

$$= 0.005 \times \frac{d}{dt}(i_0 \sin \omega t)$$

$$= 0.005 \times i_0 \omega \cos \omega t \qquad [\text{for } e_{max}, \cos \omega t = 1]$$

$$\therefore \quad e_{max} = 0.005 \times 10 \times 100\pi$$

$$= 5\pi$$

3. (a) emf, $e_2 = M\left(\frac{di_1}{dt}\right) = i_2 R_2$

$\therefore$ Current $\left(\frac{di_1}{dt}\right) = \frac{i_2 R_2}{M} = \frac{(0.4)(5)}{0.5} = 4 \text{ As}^{-1}$

5. (d) Average emf, $e = M\frac{di}{dt} = 0.09 \times \frac{20}{0.006} = 300$ V

6. (*a*) As, $|e| = M\frac{di}{dt} \Rightarrow 8\times10^{-3} = M\times 3$

Mutual inductance, $M = 2.66$ mH

7. (*a*) We have, $M = K\sqrt{L_1L_2}$

For perfect coupling, $K = 1 \Rightarrow M_{12} = M_{21}$

8. (*a*) Mutual inductance between coils is

$$M = K\sqrt{L_1L_2}$$

$$\Rightarrow \quad M = 1\sqrt{2\times10^{-3}\times8\times10^{-3}} \qquad [\because K = 1]$$

$$= 4\times10^{-3} = 4 \text{ mH}$$

9. (*c*) Mutual inductance between two coils,

$$M = -\frac{e_2}{(di_1/dt)} = -\frac{e_1}{(di_2/dt)}$$

Also, $e_1 = -L_1\frac{di_1}{dt}$ and $e_2 = -L_2\frac{di_2}{dt}$

$$\therefore \quad M^2 = \frac{e_1e_2}{\left(\frac{di_1}{dt}\right)\left(\frac{di_2}{dt}\right)} = L_1L_2 \Rightarrow M = \sqrt{L_1L_2}$$

10. (*b*) Growth of current in the circuit is given by

$$i = i_0(1 - e^{-Rt/L})$$

where, i_0 is peak value of current and

$$i_0 = \frac{5}{5} = 1\text{A}$$

$$\therefore \quad i = 1(1 - e^{-5\times2/10}) = (1 - e^{-1})\text{ A}$$

11. (*b*) In case of growth of current in a L-R circuit, the current in the circuit grows exponentially with time 0 to the maximum value at, $i_0 = E/R$.

12. (*c*) Time interval, during which the current in an inductive circuit rises to 63% of its maximum value is defined as time constant.

A Taking it together

1. (*d*) We have, $\tau_c = CR$, $\tau_L = \frac{L}{R}$ and $\omega = \frac{1}{\sqrt{LC}}$

2. (*b*) The self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length, filled the inside of the solenoid with a material of relative permeability (*e. g.*, soft iron, which has a high value of relative permeability) is given by

$$L = \mu_r\mu_0n^2Al = \frac{\mu_r\mu_0N^2A}{l}$$

3. (*b*) Induced emf, $e = -\frac{d\phi}{dt}$

But, $e = iR$ and $i = \frac{dq}{dt}$

$$\therefore \quad \frac{dq}{dt}R = -\frac{d\phi}{dt} \text{ or } dq = -\frac{d\phi}{R}$$

So, induced charge in a coil is independent of time taken to change the flux.

4. (*b*) There will be self-induction effect when soft iron core is inserted in the coil. So, intensity of bulb increases.

5. (*b*) The magnitude of induced emf is directly proportional to the rate of change of magnetic flux. Also, induced charge is $dq = -\frac{d\phi}{R}$, which is independent of time.

6. (*a*) Induced current is given by

$$i = \frac{e}{R} = -\frac{N(d\phi/dt)}{R} = -\frac{N}{R}\left(\frac{d\phi}{dt}\right)$$

$$= \frac{10\times10^8\times10^{-4}\times10^{-4}\times10}{20} = 5\text{ A}$$

7. (*c*) In L-R circuit current grows exponentially.

So, B_2 lights up earlier and finally B_1 shines brighter than B_2.

8. (*c*) Apply Lenz's law, then loop B is repelled by loop A.

9. (*d*) Induced emf, $e = -N\frac{d\phi}{dt}$

$$|e| = N\frac{d\phi}{dt} = N\frac{d}{dt}(BA\cos\theta) = N\frac{dB}{dt}\cdot A\cos\theta$$

$$= 500\times1\times(10\times10^{-2})^2\cos0°$$

$$= 5\text{ V}$$

10. (*b*) Induced emf, $e = -\frac{d\phi}{dt}$

$$|e| = \frac{d}{dt}(BA) = A\frac{dB}{dt} = 2\times\frac{(4-1)}{2} = 3\text{ V}$$

12. (*c*) $\phi = 0$. Therefore, $\Delta\phi = 0$.

13. (*a*) The magnet gets repelled. Therefore, north pole is formed on the top side and hence current will flow in anti-clockwise direction.

14. (*c*) Current in the circuit, $i = V/R = 10/2 = 5\text{A}$

$\therefore$ Magnetic energy stored in the coil,

$$U = \frac{1}{2}Li^2 = \frac{1}{2}\times2\times(5)^2 = 25\text{ J}$$

15. (*b*) Time constant, $\tau_L = \frac{L}{R} = \frac{1}{2}$

We have, $i = i_0(1 - e^{-t/\tau_L})$

$$\therefore \quad \frac{i_0}{i} = \frac{1}{1 - e^{-t/\tau_L}}$$

Substituting, $t = 1$s and $\tau_L = \frac{1}{2}$ s, we get

$$\frac{i_0}{i} = \frac{e^2}{e^2 - 1}$$

16. (*d*) For a capacitor, $i = \left(\frac{dq}{dt}\right) = C\cdot\left(\frac{dV}{dt}\right)$

i.e., i = constant, if $\frac{dV}{dt}$ = constant

$\therefore$ Capacitance, $C = \frac{i}{\left(\frac{dV}{dt}\right)} = \frac{1}{2} = 0.5\text{ F}$

17. (*a*) Charge induced in coil is given as

$$dq = \frac{d\phi_B}{R} = i\,dt = \text{Area under } i\text{-}t \text{ graph}$$

$$\therefore \quad d\phi_B = (\text{Area under } i\text{-}t \text{ graph})R$$

$$= \frac{1}{2} \times 4 \times 0.1 \times 10 = 2 \text{ Wb}$$

19. (*d*) We have, $\Delta q = -\frac{N}{R}\Delta\phi$ and $\Delta\phi = \Delta B \cdot A\cos\theta$

$$\therefore \quad 32 \times 10^{-6} = -\frac{100}{(160+40)}(0-B) \times \pi \times (6\times10^{-3})^2 \times \cos 0°$$

$\therefore$ Intensity of magnetic field, $B = 0.566$ T

20. (*d*) Induced emf,

$$e = \left|\frac{-NBA(\cos\theta_2 - \cos\theta_1)}{\Delta t}\right|$$

$$= \left|-\frac{800 \times 4 \times 10^{-5} \times 0.05(\cos 90° - \cos 0°)}{0.1}\right| = 0.016 \text{ V}$$

21. (*a*) $\therefore$ Magnetic flux, $\phi = B \cdot A$

$\therefore$ Change in flux, $d\phi = B \cdot dA$

$$= 0.05\,(101 - 100) \times 10^{-4} = 5 \times 10^{-6} \text{ Wb}$$

$$\therefore \quad \text{Charge, } dq = \frac{d\phi}{R} = \frac{5\times10^{-6}}{2} = 2.5 \times 10^{-6} \text{ C}$$

22. (*b*) For growth of current in L-R circuit, current is given by

$$i = i_0(1 - e^{-Rt/L})$$

$$\therefore \quad \frac{di}{dt} = -i_0\left(-\frac{R}{L}\right)e^{-Rt/L} = \frac{i_0 R}{L}e^{-Rt/L}$$

At $\quad \frac{di}{dt} = \frac{i_0 R}{L} = \frac{E}{L} \Rightarrow 4 = \frac{E}{20}$

$\therefore \quad E = 20 \times 4 = 80$ V

23. (*d*) Peak value of emf generated in generator is given by

$$e_0 = \omega NBA$$

$$= (2\pi\nu)NBA$$

$$= 2 \times 3.14 \times 100 \times 5000 \times 0.2 \times 0.25$$

$$= 157 \text{ kV}$$

24. (*d*) emf, $e = Bl^2\pi\nu$

$$= 0.4 \times 10^{-4} \times (0.5)^2 \times (3.14) \times \frac{120}{60} = 6.28 \times 10^{-5} \text{V}$$

25. (*d*) If electron is moving from left to right, the flux linked with the loop (which is into the page) will first increase and then decrease as the electron passes through it. So the induced current in the loop will be first anti-clockwise and will change direction as the electron passes by.

26. (*d*) **B**, **v** and **L** should be mutually perpendicular.

27. (*c*) Induced emf, $e = Bvl$

$\therefore \quad e \propto v$

Also, $\quad v = 0 + gt$

$\therefore \quad e \propto gt$

So, potential difference between its two ends will increase with time.

28. (*a*) In the given circuit, three inductances are in parallel, their equivalent inductance is given by

$$\frac{1}{L_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \text{ or } L_{eq} = \frac{3}{3} = 1\text{H}$$

29. (*d*) $\therefore$ In parallel, $L_P = \frac{L_1 L_2}{L_1 + L_2}$

$$(L_1 + L_2)1.5 = L_1 L_2$$

and in series, $\quad L_S = L_1 + L_2 = 8$

$\therefore \quad L_1 L_2 = 12$

$$(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1L_2$$

$$L_D = \sqrt{(8)^2 - 4 \times 12}$$

$$L_D = 4\text{H}$$

30. (*c*) Time constant, $\tau = \frac{L}{R}$

It is the time interval during which the current after opening an inductive circuit falls to 37% of its maximum value.

31. (*c*) Time constant, $\tau = \frac{L}{R} = 2.0 \times 10^{-3}$ s

and $\quad \frac{L}{R+90} = 0.5 \times 10^{-3}$ s

Solving these two equations, we get

$$L = 60 \text{ mH} \quad \text{and} \quad R = 30\,\Omega$$

32. (*c*) Four resistances form a balanced Wheatstone bridge. Therefore, energy stored in the coil is zero.

33. (*d*) The whole current will pass through the inductor. So, current in 10 Ω resistor is zero.

34. (*b*) Power, $\quad P = Fv = ilBv$

$$= \left(\frac{Bvl}{R}\right)lBv = \frac{B^2l^2v^2}{R}$$

35. (*c*) The magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

$$\phi = \mathbf{B} \cdot \mathbf{A}$$

Here, $\quad A = L^2\hat{\mathbf{k}}$ and $B = B_0\,(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T

$\therefore \quad \phi = \mathbf{B} \cdot \mathbf{A} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot L^2\hat{\mathbf{k}} = 4B_0L^2$ Wb

36. (*a*) $\therefore$ Magnetic flux,

$$\phi = BA\cos\theta = 0.2\sin(300t) \times 5 \times 10^{-4} \times \cos 60°$$

$$= 5 \times 10^{-5}\sin(300t)$$

$$\therefore \text{ emf, } e = \frac{d\phi}{dt} = 5 \times 10^{-5} \times 300\cos(300t)$$

$$= 15 \times 10^{-2}\cos\left(300 \times \frac{\pi}{900}\right)$$

$$= 1.5 \times 10^{-2}\cos(\pi/3) = 0.75 \times 10^{-2} = 7.5 \times 10^{-3} \text{ V}$$

37. (*c*) Since, the magnetic field is uniform, therefore there will be no change in flux, hence no current will be induced.

39. (*b*) Charge, $\Delta Q = \frac{\Delta\phi}{R} = \frac{n \times BA}{R}$

$\Rightarrow$ Magnetic flux density,

$$B = \frac{\Delta Q \cdot R}{nA} = \frac{2\times10^{-4} \times 80}{40 \times 4 \times 10^{-4}} = 1 \text{ Wb/m}^2$$

40. (*b*) We have, $L \propto N^2$

$$\frac{L_B}{L_A} = \left(\frac{n_B}{n_A}\right)^2$$

Self-inductance, $L_B = \left(\frac{500}{600}\right)^2 \times 108 = 75 \text{ mH}$

42. (*a*) Mutual inductance between two coils depend on their degree of flux linkage *i.e.*, the fraction of flux linked with one coil which is, when some current passes through the other coil. In figure (A) two coils with their planes are parallel. In this situation, maximum flux passes.

43. (*d*) During growth of current in the coil,

$$i = i_0(1 - e^{-Rt/L})$$

For $i = \frac{i_0}{2}$

$$\Rightarrow \quad \frac{i_0}{2} = i_0 (1 - e^{-Rt/L})$$

$$\therefore \quad t = 0.693 \frac{L}{R} = 0.693 \times \frac{300 \times 10^{-3}}{2} = 0.1 \text{ s}$$

44. (*c*) Growth of current in the circuit is

$$i = i_0(1 - e^{-Rt/L})$$

$$\Rightarrow \quad \frac{di}{dt} = \frac{d}{dt} i_0 - \frac{d}{dt}(i_0 e^{-Rt/L})$$

$$\therefore \quad \frac{di}{dt} = 0 - i_0 \left(-\frac{R}{L}\right) e^{-Rt/L} = \frac{i_0 R}{L} e^{-Rt/L}$$

Initially, $t = 0$,

$$\therefore \quad \frac{di}{dt} = \frac{i_0 R}{L} = \frac{E}{L} = \frac{5}{2} = 2.5 \text{As}^{-1}$$

45. (*a*) Just after pressing the key K, the inductor offers infinite resistance. So, net resistance will be 14Ω.

$$\therefore \quad I = V/R = 5/7\text{A}$$

46. (*a*) At $t = 0$, current will flow from the capacitor and at $t = \infty$ current flows from the inductor. So, current in both cases will be 1.5 A.

47. (*a, b, c*) We have, $V = 2t$

$$\therefore \quad V = L \cdot \frac{di}{dt} = 2t \text{ or } 2 \cdot \frac{di}{dt} = 2t \text{ or } \frac{di}{dt} = t$$

or $(di) = t\,(dt)$

On integrating, we get $i = \frac{t^2}{2}$

i.e., i-t graph is a parabola.

At $t = 2$ s, $i = 2$ A

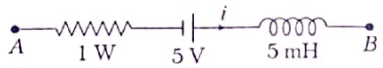
$\therefore$ Potential energy, $U = \frac{1}{2} Li^2 = \frac{1}{2} \times 2 \times 4 = 4 \text{ J}$

$$\frac{dU}{dt} = Li\left(\frac{di}{dt}\right) = (2)\left(\frac{t^2}{2}\right)(t) = t^3$$

At $t = 1$

Potential energy, $\frac{dU}{dt} = 1 \text{ Js}^{-1}$

48. (*c*) Applying Kirchhoff's second law to above circuit,



$$V_A - iR + 15 - L\frac{di}{dt} = V_B$$

or $V_A - 5 \times 1 + 15 - (5 \times 10^{-3})(-10^3) = V_B$

$$\therefore \quad V_B - V_A = 15 \text{ V}$$

49. (*c*) At $t = 0$, induced emf e is maximum and is equal to applied emf, but current i is zero.

After lapse of time, current through the circuit increases but induced emf decreases.

50. (*b*) Two coils are said to be magnetically coupled, if full or a part of the flux produced by one link with the other. Let L_1 and L_2 be the self-inductances of the coils and M be their mutual inductances, then

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

When 100% flux produced by one coil links with the other, then mutual inductance between the two is maximum and is given by

$$M = \sqrt{L_1 L_2}$$

51. (*b*) In position II, change in flux = 0, emf = 0.

In position I and III, $e = Bav$.

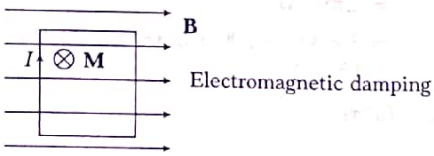
In position I, flux is increasing, hence anti-clockwise emf.

If position III, flux is decreasing, hence clockwise emf.

52. (*b*) Induced emf, $|e| = \frac{d\phi}{dt} = \frac{BdS}{dt}$

Now, as the square loop and rectangular loop move out of magnetic field, $\frac{BdS}{dt}$ is constant, therefore $|e|$ is constant. But in case of circular and elliptical loops, $\frac{dS}{dt}$ changes. Therefore, $|e|$ does not remain constant.

54. (*d*)



55. (*d*) Mutual inductance between two coils in the same plane with their centres coinciding is given by

$$M = \frac{\mu_0}{4\pi}\left(\frac{2\pi^2 R_2^2 N_1 N_2}{R_1}\right) \text{henry}$$

56. (*b*) Power, $P = \frac{e^2}{R}$, hence $e = -\left(\frac{d\phi}{dt}\right)$, where $\phi = NBS$

$$\therefore \quad e = -NS\left(\frac{dB}{dt}\right). \text{ Also, } R \propto \frac{l}{r^2}$$

where, R = resistance, r = radius, l = length

$$\therefore \quad P \propto \frac{N^2 r^2}{l} \Rightarrow \frac{P_1}{P_2} = 1$$

57. (*b*) As the magnet moves towards the coil, the magnetic flux increases (non-linearly). Also, there is a change in polarity of induced emf when the magnet passes on to the other side of the coil.

58. (*b*) emf, $e \propto -\frac{di}{dt}$.

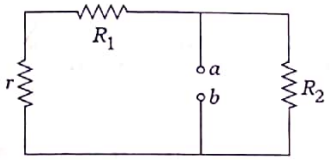
59. (*b*) $i = i_0\left(1 - e^{-\frac{R}{L}t}\right)$. At $t = 0$, $i = 0$.

At $t >> \tau (= L/R)$, $i = i_0$.

60. (*c*) Since, you are decreasing current in the circuit by increasing resistance of the circuit.

Induced emf across inductor will support 20 V battery. Hence, net emf of the circuit is greater than 20 V or current in the circuit is more than 4 A.

61. (*c*) After short-circuiting the battery.



Resistance, $R_{ab} = \frac{R_2 (R_1 + r)}{R_2 + R_1 + r}$

$\therefore$ Time constant, $\tau_L = \frac{L}{R_{net}} = \frac{L (R_1 + R_2 + r)}{(R_1 + r) R_2}$

62. (*d*) Magnetic lines on circular loop, due to current in wire AB are tangential to its plane.

$\therefore$ Magnetic flux, $\phi_B = 0$ or induced emf, $\frac{d\phi_B}{dt} = 0$

63. (*a*) Induced emf, $|e| = \frac{\Delta\phi}{\Delta t} = \frac{(2-1)(0.01)}{10^{-3}} = 10\,\text{V}$

$\therefore$ Current, $i = \frac{e}{R} = 10^3\,\text{A}$

Heat produced, $H = eit = (10)(10^3)(10^{-3}) = 10\,\text{J}$

64. (*d*) Induced emf between the axle and the rim is given by

$e = \frac{1}{2} Bl^2\omega = \pi l^2 fB$ [as $\omega = 2\pi f$]

$= 0.4 \times 10^{-4} \times (0.5)^2 \times (3.14) \times \frac{120}{60} = 6.28 \times 10^{-5}$ V

65. (*d*) As, potential difference,

$V = Bvl = (2.0 \times 10^{-4})\left(360 \times \frac{5}{18}\right)(50) = 1.0$ V

66. (*a*) If the loop remains inside the region of magnetic field, no current will be induced. But, if it moves out of the plane, then the induced current is given by $i = \frac{Blv}{R}$ and its direction will be clockwise (as per Lenz's law).

67. (*b*) As, Potential difference, $e = Bvl$

$= (0.2 \times 10^{-4})(20)(1)$ V

$= 0.4$ mV

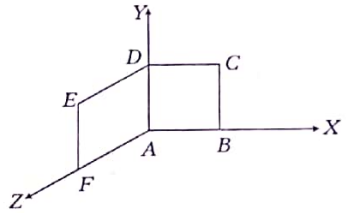
68. (*c*) $M_{12} = M_{21}$, when rate of change of current is doubled, induced emf also becomes two times.

Thus, emf produced across the first coil is 6 mV.

69. (*b*) Here, loop $ABCDA$ lies in XY-plane whose area vector $\mathbf{A}_1 = L^2\hat{\mathbf{k}}$, whereas loop $ADEFA$ lies in YZ-plane whose area vector $\mathbf{A}_2 = L^2\hat{\mathbf{i}}$.

Also, the magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

$\phi = \mathbf{B} \cdot \mathbf{A}$



$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 = (L^2\hat{\mathbf{k}} + L^2\hat{\mathbf{i}})$

and $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})$ T

Now, $\phi = \mathbf{B} \cdot \mathbf{A} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}}) \cdot (L^2\hat{\mathbf{k}} + L^2\hat{\mathbf{i}}) = 2B_0L^2$ Wb

70. (*b*) Two emfs are induced in the closed circuit each of value Blv. Their two emfs are additive. So, $E_{net} = 2Blv$.

71. (*a, c*) $e = -\frac{d\phi}{dt} = -A \cdot \left(\frac{dB}{dt}\right) = -A$ (slope of B-t graph)

At $t = 2$, slope is zero and it changes its sign.

72. (*d*) Magnetic flux,

$\phi = NAB = (25)(\pi)(1.8 \times 10^{-2})^2[0.2t - 0.05t^2]$

$= 0.025(0.2t - 0.05t^2)$

Induced emf, $e = \left|\frac{d\phi}{dt}\right| = |0.025\,(0.2 - 2 \times 0.05\,t)|$

At $t = 3$ s, $e = 0.0025$ V

$i = \frac{e}{R} = 0.0016$ A

$\therefore$ Power dissipation, $P = ei = 4 \times 10^{-6}$ W $= 4\,\mu$W

73. (*a, c*) In both the cases (a) and (c) the induced current in the ring will be in such a direction that it attracts the magnet.

74. (*b*) $e = Bvl = (0.1)(1)(0.1) = \frac{1}{100}$ V

Net resistance $= 1 + \frac{2 \times 3}{2 + 3} = \frac{11}{5}\Omega$

$\therefore$ Current, $i = \left(\frac{1/100}{11/5}\right) = \frac{1}{220}$ A

75. (*a*) From the figure, $V_a - iR - L\frac{di}{dt} = V_b$

$\therefore$ $V_a - V_b = iR + L\frac{di}{dt}$

According to given conditions,

$8 = 2R + L$ ...(i)

$4 = 2R - L$ ...(ii)

Solving these two equations, we get

$R = 3\,\Omega$ and $L = 2$ H

76. (*b*) $V = L\frac{di}{dt}$ or $V \propto L\left(\text{as } \frac{di}{dt} \to \text{same}\right)$

$\therefore \quad \frac{V_1}{V_2} = \frac{L_1}{L_2} = 4, \qquad P = Vi = \text{constant}$

$\therefore \quad i \propto \frac{1}{V}$ or $\frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4} \Rightarrow W = \frac{1}{2}Li^2$

$\therefore \quad \frac{W_1}{W_2} = \frac{L_1}{L_2}\left(\frac{i_1}{i_2}\right)^2 = (4)\left(\frac{1}{16}\right) = \frac{1}{4}$

$\because \quad \frac{i_1}{i_2} \neq 4$

Thus, wrong option is (b).

77. (*c*) Induced emf, $E = -L\frac{di}{dt}$

or induced emf $\propto$ – slope of i-t graph. So, graph (c) is correct.

78. (*d*) Energy, $U = \frac{1}{2}Li_0^2 = \frac{1}{2}L\left(\frac{V}{R}\right)^2$

$$= \frac{1}{2} \times 50 \times 10^{-6}\left(\frac{5}{0.5}\right)^2 \text{J} = 2.5\,\text{mJ}$$

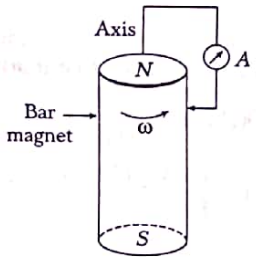
Now, after switching off the battery this energy is dissipated in coil (resistance = 0.5 Ω) and resistance (10 Ω) in the ratio of their resistances ($H = i^2Rt$ or $H \propto R$). Therefore, heat generated in the coil.

$$\left(\frac{5}{0.5}\right)^2\left(\frac{0.5 \times 10}{0.5 + 10}\right) 2.5\,\text{mJ} = 0.12\,\text{mJ}$$

79. (*b*) Power, $P = (i_0)^2R$, *i.e.*, $(i_0)^2 = \frac{P}{R}$ and $I = \frac{L}{R}$

Total heat produced, $U = \frac{1}{2}Li_0^2 = \frac{1}{2}(\tau R)\left(\frac{P}{R}\right) = \frac{1}{2}P\tau$

80. (*b*) When cylindrical bar magnet is rotated about its axis, no change in flux linked with the circuit takes place, consequently no emf induces and hence no current flows through the ammeter A.



81. (*d*) When A stops moving the current in B become zero, possible only if the current in A is constant. If the current in A would be variable, then there must be an induced emf (current) in B even, if A stops moving.

82. (*a*) When the current in B (at $t = 0$) is counter-clockwise and the coil A is considered above to it. The counter-clockwise flow of the current in B is equivalent to north pole of magnet and magnetic field lines are emanating upward to coil A. When coil A start rotating at $t = 0$, then the current in A is constant along clockwise direction by Lenz's rule.

83. (*a*) Here, area, $A = 10 \times 5 = 50\ \text{cm}^2 = 50 \times 10^{-4}\ \text{m}^2$

$$\frac{dB}{dt} = 0.2\ \text{Ts}^{-1} \Rightarrow R = 2\,\Omega$$

$\therefore \quad$ emf, $e = \frac{d\phi}{dt} = A \cdot \frac{dB}{dt} = 50 \times 10^{-4} \times 0.02 = 10^{-4}$ V

Power dissipated in the form of heat

$$= \frac{e^2}{R} = \frac{10^{-4} \times 10^{-4}}{2} = 0.5 \times 10^{-8}\,\text{W} = 5 \times 10^{-9}\,\text{W} = 5\,\text{nW}$$

84. (*d*) Cross ⊗ magnetic field passing from the closed loop is increasing. Therefore, from Lenz's law, induced current will produce dot ⊙ magnetic field. Hence, induced current is anti-clockwise.

In that case, $k = 1$(unity)

85. (*a*) Only half area of ring will be considered.

$\therefore \quad \phi_B = BS = B \cdot \frac{\pi r^2}{2}$

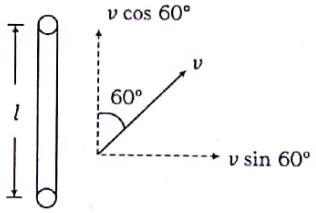
$\therefore \quad e = \frac{d\phi_B}{dt} = \frac{\pi r^2}{2} \cdot \frac{dB}{dt} = \frac{\pi(0.1)^2}{2} \times 0.01 = \frac{\pi}{2} \times 10^{-4}$ V

$\therefore \quad i = \frac{e}{R} = \frac{\frac{\pi}{2} \times 10^{-4}}{4} = \frac{\pi}{8} \times 10^{-4} = 3.9 \times 10^{-5}\text{A}$

86. (*a*) $\therefore$ emf, $|\bar{e}| = \frac{\Delta\phi}{\Delta t} = \frac{BS - 0}{\Delta t} = \frac{2 \times 10^{-4} \times \pi \times (0.1)^2}{2 \times 2}$

$\Rightarrow \quad |\bar{e}| = 1.57 \times 10^{-6}$ V

87. (*b*)

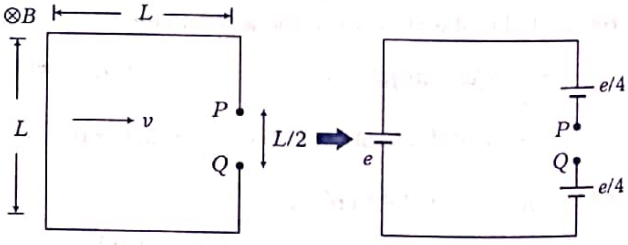


Due to $v \cos 60°$, $e = 0$

Due to $v \sin 60°$, $e = B(v \sin 60°)l$

$$= \frac{\sqrt{3}}{2}Bvl = \frac{\sqrt{3}}{2} \times 1 \times 0.2 \times 0.1 = 17 \times 10^{-3}\,\text{V}$$

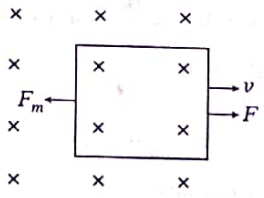
88. (*c*)



$e = BLv$

$$V_P + \frac{e}{4} - e + \frac{e}{4} = V_Q \Rightarrow V_P - V_Q = \frac{e}{2} = \frac{BLv}{2}$$

89. (*b*)



Induced emf $= Bvl$

Induced current, $i = \dfrac{Bvl}{R}$

Magnetic force on loop, $F_m = Bil = \dfrac{B^2v l^2}{R}$

Since, loop is moving with constant speed,

$\Rightarrow \quad F = F_m$

Input power, $P = \mathbf{F}\cdot\mathbf{v} = Fv = \dfrac{B^2v^2l^2}{R}$

P *versus* v graph will be like,

$y \propto x^2$ (parabola, open upward)

90. (*a*) Area of square loop, $S = 10\text{ cm} \times 10\text{ cm}$

$\Rightarrow S = 100\text{ cm}^2 = 100\times10^{-4}\text{ m}^2 = 10^{-2}\text{m}^2$

Initial magnetic flux linked with loop,

$\phi_{B_1} = B_1 S\cos\phi = 0.1\times10^{-2}\times\cos 45°$

$$= \frac{0.1\times10^{-2}\times1}{\sqrt{2}} = \frac{10^{-3}}{\sqrt{2}}\text{ Wb}$$

Final magnetic flux linked with loop,

$\phi_{B_2} = 0$ Wb $\quad [\because B_2 = 0]$

The induced emf in the loop, $\varepsilon = -\dfrac{d\phi}{dt} = -\dfrac{(\phi_2 - \phi_1)}{t}$

$$= -\frac{\left(0 - \frac{10^{-3}}{\sqrt{2}}\right)}{0.7} = \frac{10^{-3}}{0.7\times\sqrt{2}} \cong 10^{-3}\text{ V}$$

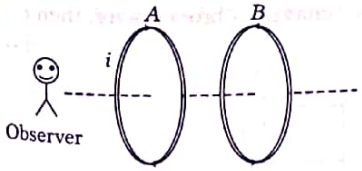
The induced current in the loop,

$$I = \frac{\varepsilon}{R} = \frac{10^{-3}\text{V}}{1\Omega} = 10^{-3}\text{A} = 1.0\text{ mA}$$

91. (*c*) At low frequency of 1 to 2 Hz also, oscillations may be observed.

92. (*a*) If bar magnet is falling vertically through the hollow region of long vertical copper tube, then the magnetic flux linked with the copper tube (due to 'non-uniform' magnetic field of magnet) changes and eddy currents are generated in the body of the tube by Lenz's law. The eddy currents oppose the falling of the magnet which, therefore experiences a retarding force. The retarding force increases with increasing velocity of the magnet and finally equals the weight of the magnet. The magnet, then attains a constant final terminal velocity, *i.e.*, magnet ultimately falls with zero acceleration in the tube.

93. (*d*)

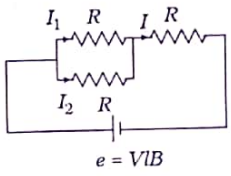


If current through A increases, magnetic field ($\times$) linked with coil B increases. Hence, anti-clockwise current induces in coil B. As shown in figure both the currents produce repulsive effect.

94. (*c*) As the emf induced $= B_1vl - B_2vl$

$$= \frac{\mu_0 I}{2\pi(x - a/2)}lv - \frac{\mu_0 I}{2\pi(x + a/2)}lv \propto \frac{1}{(2x-a)(2x+a)}$$

95. (*c*) Circuit can be reduced as

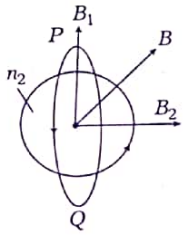


$$I = \frac{e}{3R/2} = \frac{2vlB}{3R}$$

$\Rightarrow \quad I_1 = I_2 = \dfrac{I}{2} = \dfrac{vlB}{3R}$

96. (*c*) Magnetic field at P, $B_1 = \dfrac{\mu_0 I_2}{2R}$

$$= \frac{4\pi\times10^{-7}\times4}{2\times0.02\pi} = 4\times10^{-5}\text{ Wb/m}^2$$



Magnetic field, at Q,

$$B_2 = \frac{\mu_0 I_1}{2R} = \frac{4\pi\times10^{-7}\times3}{2\times0.02\pi} = 3\times10^{-5}\text{ Wb/m}^2$$

$\therefore \quad B = \sqrt{B_1^2 + B^2{}_2}$

$$= \sqrt{(4\times10^{-5})^2 + (3\times10^{-5})^2} = 5\times10^{-5}\text{ Wb/m}^2$$

97. (*d*) During decay of current,

$$i = i_0 e^{-\frac{Rt}{L}} = \frac{E}{R}e^{-\frac{Rt}{L}} = \frac{100}{100}e^{-\frac{100\times10^{-3}}{100\times10^{-3}}} = \frac{1}{e}\text{A}$$

98. (*b*) The self-inductance of a coil, $L = \mu_0 n^2 Sl$, where μ_0 = permeability of air,

n = number of turns per unit length,

S = area of cross-section and

l = length of the solenoid

This depends on the geometry of the inductor such as cross-sectional area, length and number of turns and not on the material, even if it is made of a superconducting material. If the superconductor is below the critical temperature, then the current will continuously flow and the inductance may not have the property of inductance any more.

$\therefore \quad L_1 = L_2 L_3 = 0$

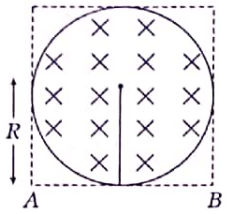
99. (*d*) Current, $I = \dfrac{\text{Potential difference}}{\text{Resistance}}$

$\therefore$ Potential difference $= 2\times12 = 24\text{ V} = Bvl$

Here, $l = AD\sin 37° = 0.3\times\dfrac{3}{5} = 0.18\text{m}$

$\therefore$ Velocity, $v = \dfrac{24}{Bl} = \dfrac{24}{4\times0.18} = \dfrac{100}{3}\text{ ms}^{-1}$

100. (*d*) Induced emf, $V_{BA} = \int \mathbf{E}\cdot d\mathbf{l} = \frac{1}{4}\left(\frac{d\phi}{dt}\right)$



$$= \frac{1}{4}(\pi R^2)\cdot\frac{dB}{dt} = \frac{\pi R^2\alpha}{4} \quad [\because \phi = B\cdot A \text{ and } \frac{dB}{dt} = \alpha]$$

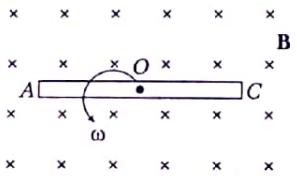
101. (*a*) According to Fleming's right hand rule, P is at higher potential and Q is at lower potential. Therefore, A is positively charged and B is negatively charged.

Also, charge, $Q = CV = C(Bvl)$

$$= 10\times10^{-6}\times4\times2\times1 = 80\ \mu C$$

$\therefore$ $q_A = 80\ \mu C$ and $q_B = -80\ \mu C$

102. (*c*) For rotating rod, induced emf, $v = \frac{1}{2}Bl^2\omega$



For part AO, $V_{OA} = V_O - V_A = \frac{1}{2}Bl^2\omega$

For part OC, $V_{OC} = V_O - V_C = \frac{1}{2}B(3l)^2\omega$

$\therefore$ $V_A - V_C = 4Bl^2\omega$

103. (*b*) In L-R circuit, for growth of current,

$$i = i_0(1-e^{-Rt/L}) \text{ or } \frac{3}{4}i_0 = i_0(1-e^{-t/\tau_L}) \text{ (given)}$$

$$\text{(where, } \tau_L = \frac{L}{R} = \text{time constant)}$$

$\therefore$ $e^{-t/\tau_L} = 1-\frac{3}{4} = \frac{1}{4}$

$$e^{t/\tau_L} = 4 \text{ or } \frac{t}{\tau_L} = \log_e 4$$

$$\tau_L = \frac{t}{\log_e 4} = \frac{4}{2\log_e 2} \Rightarrow \tau_L = \frac{2}{\log_e 2}\text{ s}$$

104. (*d*) As the coil is rotated, angle θ (angle which is normal to the coil makes with **B** at any instant t) changes, therefore magnetic flux ϕ linked with the coil changes and hence an emf is induced in the coil. At this instant t, if e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NSB\cos\omega t)$$

where, N is number of turns in the coil.

or $e = -NSB\frac{d}{dt}(\cos\omega t) = -NSB(-\sin\omega t)\,\omega$

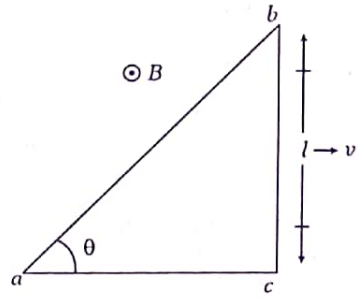
or $e = NSB\omega\sin\omega t$ ...(i)

The induced emf will be maximum when, $\sin\omega t$ *i.e.*, maximum

$\therefore$ $e_{max} = e_0 = NSB\omega\times1$ or $e = e_0\sin\omega t$

Therefore, e would be maximum, hence current is maximum (as $i_0 = e_0/R$), when $\theta = 90°$ *i.e.*, normal to plane of coil is perpendicular to the field or plane of coil is parallel to magnetic field.

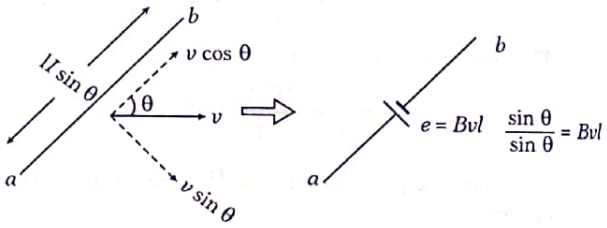
105. (*a*)

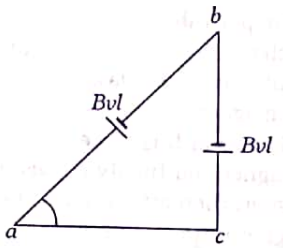


For segment bc, emf $e = Bvl$

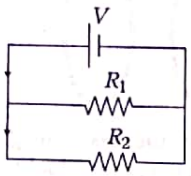
For segment ac, emf $ac: e = 0$

For segment ab, emf ab:





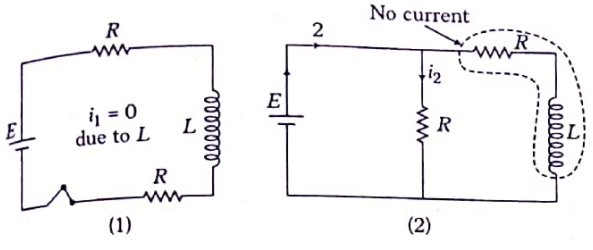
106. (*c*) At $t = 0$ inductor behaves as broken wire, then $i = \frac{V}{R_2}$.

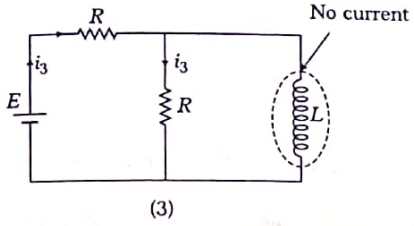


At $i = \infty$, inductor behaves as a conducting wire,

$$i = \frac{V}{R_1R_2/(R_1+R_2)} = \frac{V(R_1+R_2)}{R_1R_2}$$

107. (a) (i) Just before closing the switch,

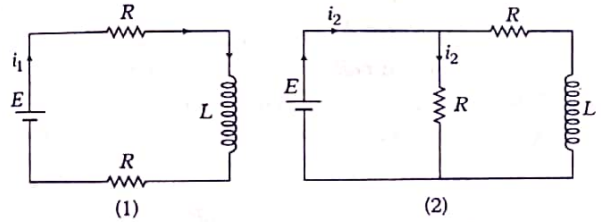




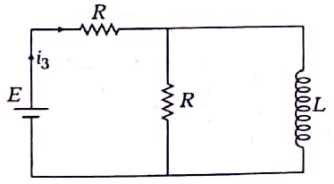
$$i_1 = 0,\ i_2 = \frac{E}{R}, i_3 = \frac{E}{2R},$$

So, $i_2 > i_3 > i_1 [i_1 = 0]$.

(ii) After a long time closing the switch,



$R_{eq} = 2R$ $\qquad$ $R_{eq} = \frac{R}{2}$



$R_{eq} = 2R$

As, $\quad i = \frac{E}{R}$

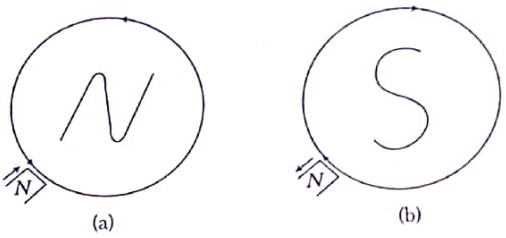
$\Rightarrow \quad i_1 = E/2R, i_2 = 2E/R, i_3 = E/R$

Therefore, $i_2 > i_3 > i_1$.

108. (d) At B, flux is maximum, so from $|e| = \frac{d\phi}{dt}$ at B, $|e| = 0$

109. (a) When the north pole of a bar magnet moves towards the coil, the induced current in the coil flows in a direction such that the coil presents its north pole to the bar magnet as shown in Fig. (a).

Therefore, the induced current flows in the coil in the anti-clockwise direction. When the north pole of a bar magnet moves away from the coil, then the induced current in the coil flows in a direction such that the coil presents its south pole to the bar magnet as shown in Fig. (b).



Therefore, induced current flows in the coil in the clockwise direction.

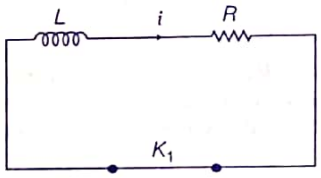
110. (b) When ring enters and leaves, then the field polarity of induced emf is opposite. Also, during the stay of ring completely in the field there is no induction.

111. (d) When switch K_1 closed for long time current through inductor, $I = \frac{E}{R} = \frac{15}{0.15 \times 10^3} = 0.1\text{A}$

When K_1 opened and K_2 is closed, then

$$i = Ie^{-t/r}$$

$$t = 1\text{ms} = 1 \times 10^{-3}\text{s}$$



$$\therefore \quad \tau_L = \frac{L}{R} = \frac{0.03}{0.15 \times 10^3} = \frac{10^{-3}}{5}$$

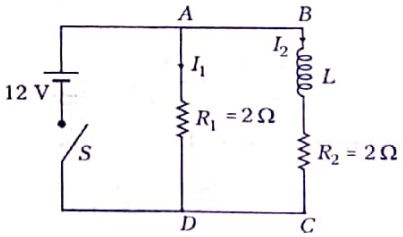
$$\therefore \quad i = 0.1e^{\left(\frac{-1\times10^{-3}}{\frac{10^{-3}}{5}}\right)} = 0.1e^{-5}$$

$$i = \frac{0.1}{e^5} = \frac{0.1}{150} = \frac{1}{15} \times 10^{-2} \times 10^3 \text{mA}$$

$$i = 0.67\text{ mA}$$

112. (d) $E(\text{across } BC) = L\frac{dI_2}{dt} + R_2 I_2$ [using Kirchhoff's law] ...(i)

$$I_2 = I_0(I - e^{-t/t_0})$$



At $t = 0$, $\quad I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\text{A}$

$$\tau_L = t_0 = \frac{L}{R} = \frac{400 \times 10^{-3}}{2\Omega} = 0.2\text{s}$$

$\therefore \quad I_2 = 6(1 - e^{-t/0.2})$ [using Eq. (i)]

Potential drop area $L = E - R_2 I_2$

$$= 12 - 2 \times 6(1 - e^{-t/0.2}) = 12e^{-t/0.2} = 12e^{-5t}\text{ V}$$

113. (*d*) When two solenoids of inductance L_0 are connected in series at large distance and current i is passed through them, then the total flux linkage $\phi_{total} = L_0 i + L_0 i$.

If L be the equivalent inductance of the system, then

$$\phi_{total} = Li$$

$$\therefore \quad Li = L_0 i + L_0 i \quad \text{or} \quad L = 2L_0$$

When solenoids are connected in series with one inside the other and senses of the turns coinciding, then there will be a mutual inductance L between them. In this case, the resultant induced emf in the coils is the sum of the emf's e_1 and e_2 in the respective coils, *i.e.*,

$$e = e_1 + e_2$$

$$= \left(-L_0 \frac{di}{dt} \pm L_0 \frac{di}{dt}\right) + \left(-L_0 \frac{di}{dt} \pm L_0 \frac{di}{dt}\right)$$

where, (+) sign is for positive coupling and (–) sign for negative coupling.

But, $$e = -L\frac{di}{dt}$$

$$\therefore \quad -L\frac{di}{dt} = -L_0\frac{di}{dt} - L_0\frac{di}{dt} \pm 2L_0\frac{di}{dt}$$

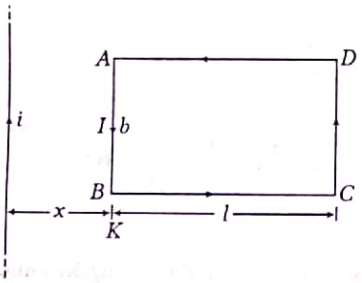
i.e., $$L = L_0 + L_0 + 2L_0$$

$$= 4L_0 \qquad \text{[for positive coupling]}$$

When solenoids are connected in series with one inside the other with senses of the turn opposite, then there is a negative coupling.

So, $$L = L_0 + L_0 - 2L_0 = 0$$

114. (*c*) We can show the situation as



Since, loop is moving away from the wire, so the direction of current in the loop will be as shown in the figure.

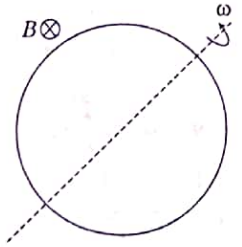
Net magnetic field on the loop due to wire,

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} - \frac{1}{l+x}\right) = \frac{\mu_0 il}{2\pi x(l+x)}$$

So, the magnitude of the emf in the loop,

$$e = vBb = \frac{\mu_0 ilvb}{2\pi x(l+x)}$$

115. (*b*)



Let at any time, normal to loop makes an angle θ with magnetic field

$$\phi_B = NBS\cos\theta = NBS\cos\omega t$$

$$e = -\frac{d\phi_B}{dt} = NBS\,\omega\sin\omega t = e_0\sin\omega t$$

$$\Rightarrow e_{max} = e_0 = NSB\omega = 1\times 0.01\times\pi(5)^2\times 10^{-4}\times\frac{80\times 2\pi}{60}$$

$$= 6.6\times 10^{-4}\,\text{V}$$

Over a long period, *i.e.* one time period, average of induced emf is

$$\bar{e} = \frac{\int_0^T e\,dt}{T} = 0$$

$$\overline{e^2} = \frac{\int_0^T e^2\,dt}{T} = \frac{e_0^2}{2} = 2.0\times 10^{-7}\,\text{V}^2$$

116. (*c*) As, $$E\,l = \frac{d\phi_B}{dt}$$

or $$E\,(2\pi R) = \pi R^2\cdot\frac{dB}{dt} = \pi R^2\,(8t)$$

$$\therefore \quad E = 4Rt$$

$$F = qE = 4qRt \qquad \text{[tangential]}$$

$$\tau_F = 4qR^2 t$$

$$\tau_f = (\mu mgR)$$

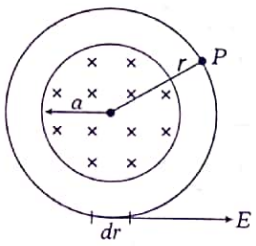
When $\tau_F > \tau_f$, ring will start rotating.

At $t = 2$ s,

$$8qR^3 = \mu mgR$$

$$\therefore \text{ Coefficient of friction, } \mu = \frac{8qR}{mg}$$

117. (*b*)



Draw a concentric circle of radius r. The induced electric field (E) at any point on the circle is equal to that at P.

For this circle, induced emf

$$e = \oint \mathbf{E}\cdot d\mathbf{l} = \left|\frac{d\phi}{dt}\right| = S\left|\frac{dB}{dt}\right|$$

$$\therefore \quad E = \oint dl = \pi a^2\left|\frac{dB}{dt}\right| \qquad [\text{but } \oint dl = 2\pi r]$$

$$\therefore \quad E\times(2\pi r) = \pi a^2\left|\frac{dB}{dt}\right|$$

$$\therefore \quad E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

$$\Rightarrow \quad E \propto \frac{1}{r}$$

118. (*d*) When switch S is closed magnetic field lines passing through Q increases in the direction from right to left.

So, according to Lenz's law, induced current in Q, *i.e.*, I_{Q_1} will flow in such a direction, so that the magnetic field lines due to I_{Q_1} passes from left to right through Q. This is possible when I_{Q_1} flows in anti-clockwise direction as seen by E. Opposite is the case when switch S is opened, *i.e.*, I_{Q_2} will be clockwise as seen by E.

119. (*a*) As the north pole approaches, a north pole is developed at the face, *i.e.*, the current flows anti-clockwise. Finally, when it completes the oscillation, no emf is present. Now, south pole approaches the other side, *i.e.*, RHS, the current flows clockwise to repel the south pole.

This means the current is anti-clockwise at the LHS as before. The break occurs when the pendulum is at the extreme and momentarily stationary.

B Medical entrance special format questions

Assertion and reason

1. (*d*) Electric flux = ES and magnetic flux = BS. Both have different units.

2. (*b*) The total number of magnetic lines of force passing through an area placed in magnetic field is equal to magnetic flux linked with the area.

Mutual inductance is the phenomenon according to which on opposing emf produces. Flux in a coil as a result of change in the current or magnetic flux linked with neighbouring coil. But when two coils are coupled, in emf produced inductively addition to induced due to mutual inductance, induced emf is also produced in each of the two coils due to self-inductance.

3. (*d*) According to Lenz's law, the direction of induced emf or current in a circuit in such as to oppose the cause that produces it.

4. (*c*) (i) Mutual inductance of two coils depends on the distance between the coils and their orientation.

(ii) Mutual inductance depends on the magnetic permeability of medium between the coils or nature of material on which two coils are wound.

5. (*d*) Self-inductance does not depend upon current flowing or change in current flowing but it depends upon of turns N. Area of cross-section (A) and permeability of medium (μ).

We have, $e = -L\dfrac{di}{dt}$.

6. (*b*) According to Lenz's law, induction effects always oppose the cause.

7. (*a*) If a circuit containing a pure inductor L and resistor R in series with a battery and a key is closed, then the circuit current through the circuit rises exponentially and reaches upto a certain maximum value.

8. (*b*) If moving from left to right, current is increasing, then battery $\left(\text{induced emf} = L.\dfrac{di}{dt}\right)$ will produce right to left current *i.e.*, its positive terminal is on left hand side or $V_A > V_B$.

Further i = constant $\Rightarrow \dfrac{di}{dt} = 0$ or $V_L = 0$.

9. (*a*) $V = L\dfrac{di}{dt}$ = constant

$\therefore \quad L\,(di)$ = constant $\Rightarrow Li$ = constant

or $\quad i \propto \dfrac{1}{L}$

10. (*a*) $L = \dfrac{-e}{di/dt} \Rightarrow L = 2/4 = 1/2$

On putting values, we get

$$\phi = Li \Rightarrow \phi = \frac{1}{2}\times 4 = 2\,\text{Wb}$$

11. (*d*) If a conducting rod of length l moving with a uniform velocity v perpendicular to a uniform magnetic field B, then induced emf will be $e = Bvl$.

12. (*e*) In non-uniform magnetic field, magnetic flux will be obtained by integration, but it will not vary with time.

13. (*c*) $\phi = Li \Rightarrow \phi \propto i$

15. (*d*) Since, both the loops are identical (same area and number of turns) and moving with a same speed in same magnetic field. Therefore, same emf is induced in both the coils. But the induced current will be more in the copper loop as its resistance wire be lesser as compared to that of the aluminium loop.

16. (*b*) $\phi = BA\cos\omega t$

At $\quad t = 0, \ \phi$ = maximum

$$|e| = \left|\frac{d\phi}{dt}\right| = B\,\omega A\sin\omega t$$

$|e|_{\max}$ when $\omega t = 90°$, *i.e.*, loop is rotated 90°, or it is parallel to magnetic field. At this instant $\phi = 0$.

17. (*a*) This is in accordance with Lenz's law. When magnet is above the loop, then north pole is formed and hence will repel and when it is below the loop, then it will attract the magnet.

Match the columns

1. A. Tesla $\rightarrow [MT^{-2}A^{-1}]$

B. Weber $\rightarrow [ML^2T^{-2}A^{-1}]$

C. Weber/m^2 $\rightarrow [MA^{-1}T^{-2}]$

D. Henry $\rightarrow [ML^2A^{-2}T^{-2}]$

2. V_{AD} at $t = 0 = E = 10$ V

V_{BC} at $t = 0 = E = 10$ V

V_{AB} at $t = \infty$

$$I = \frac{E}{R+r} = \frac{10}{5} = 2\,\text{A}$$

$$\therefore \quad V_{AB} = V_A - V_B = I\times R = 2\times 4 = 8\,\text{V}$$

V_{AD} at $t = \infty$

$$V_{AD} = V_A - V_D = E - Ir = 10 - (2\times 1) = 8\,\text{V}$$

3. In all of the cases, the induced current in the loop is non-zero. The direction of induced current can be determined by Lenz's law. When loop is moved towards right, then induced current in the loop is clockwise. When loop is moved towards left, then induced current in the loop is anti-clockwise.

4. When current i_2 is decreased, then flux through A decreases. According to Lenz's law, attraction occurs between A and B. Similarly, when current i_1 is increased, then loops will repel each other. As distance between the loop decreases, M decreases. Hence, loops will repel each other and current i_1 will increase, when loop 1 is moved towards loop 2.
When loop 2 is moved away, loops will attract and i_2 will increase.

5. A. The magnetic flux through the rectangular loop *abcd* increases, due to the motion of the loop into the region of magnetic field. The induced current must flow along the path *bcdab*, so that it opposes the increasing flux.

B. Due to the outward motion, magnetic flux through the triangular loop *abc* decreases due to which the induced current flows along *bacd*, so as to oppose the change in flux.

C. As the magnetic flux decreases due to motion of the irregular shaped loop *abcd* out of the region of magnetic field, the induced current flows along *cdabc*, so as to oppose change in flux.

C Medical entrance's gallery

1. (*c*) Given, number of turns of solenoid, $N = 1000$

Current, $I = 4$ A

Magnetic flux, $\phi_B = 4 \times 10^{-3}$ Wb

$\because$ Self-inductance of solenoid is given by

$$L = \frac{\phi_B N}{I} \qquad \text{...(i)}$$

Substituting the given values in Eq. (i), we get

$$L = \frac{4 \times 10^{-3} \times 1000}{4} = 1 \text{ H}$$

2. (*c*) Induced emf in the region is given by

$$|e| = \frac{d\phi}{dt}$$

where, $\phi = BA = \pi r^2 B$

$$\Rightarrow \quad \frac{d\phi}{dt} = -\pi r^2 \frac{dB}{dt}$$

Rate of change of magnetic flux associated with loop 1,

$$e_1 = -\frac{d\phi_1}{dt} = -\pi r^2 \frac{dB}{dt}$$

Similarly, e_2 = emf associated with loop 2

$$= -\frac{d\phi_2}{dt} = 0 \qquad (\because \phi_2 = 0)$$

3. (*a*) Given, $n = 500$

$$L = 50 \text{ mH} = 50 \times 10^{-3} \text{ H}$$

and $\quad i = 8 \text{ mA} = 8 \times 10^{-3}$ A

The magnetic flux linked with the coil is

$$\phi = Li = 50 \times 10^{-3} \times 8 \times 10^{-3}$$

$$= 400 \times 10^{-6} = 4 \times 10^{-4} \text{ Wb}$$

4. (*a*) The flux is given by

$$\phi = \mathbf{B} \cdot \mathbf{A}$$

$$\phi = |\mathbf{B}||\mathbf{A}| \cos\theta$$

Also, induced emf, $\varepsilon = -\frac{d\phi}{dt} = \frac{d[|\mathbf{B}||\mathbf{A}| \cos\theta]}{dt}$

$$= -|\mathbf{B}||\mathbf{A}| \frac{d(\cos\theta)}{dt} = |\mathbf{B}||\mathbf{A}| \sin\theta \cdot \omega$$

$$\Rightarrow \quad \varepsilon = BA\,\omega \cos(\theta - \pi/2)$$

5. (*b*) Given, $B = 40$ mT $= 40 \times 10^{-3}$T

Rate of shrinking of area $= 0.5 \text{ m}^2\text{s}^{-1}$

$$\Rightarrow \quad \frac{dA}{dt} = 0.5 \text{ m}^2\text{s}^{-1}$$

As, $\phi = BA \cos\theta = BA$ $\qquad [\because \cos\theta = \cos 0° = 1]$

On differentiating with w.r.t. to t,

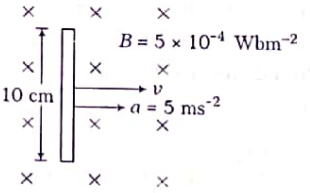
$$\frac{d\phi}{dt} = B \cdot \frac{dA}{dt} = e$$

$$\Rightarrow \quad e = \text{induced emf} = 40 \times 10^{-3} \times 0.5 = 20 \times 10^{-3} \text{V}$$

$$= 20 \text{ mV}$$

6. (*a*) Changing magnetic fields can set up current loop in nearby metal bodies and the current is called as eddy current.

7. (*b*) Consider a rod of 10 cm length is moving perpendicular to uniform magnetic field as shown below.



The emf induced in the rod, $\varepsilon = vBl$

Differentiating with respect to time t, we get

Rate of increase of induced emf $= \frac{d\varepsilon}{dt}$

$$= \frac{dv}{dt} Bl = a\,Bl$$

$$= 5 \times 5 \times 10^{-4} \times 10 \times 10^{-2}$$

$$= 250 \times 10^{-6} = 2.5 \times 10^{-4} \text{ Vs}^{-1}$$

8. (*b*) Induced emf in both the loops will be same as

$$e = \frac{-d\phi}{dt} = B\,\omega \sin\omega t$$

Since, B and ω are same for both the cases, induced emf will be same.

However, induced current is given by

$$I = \frac{e}{R}$$

So, it will depend upon the value of resistance, lower the resistance higher will be the current. Therefore, induced current is different in both the loops.

9. (*b*) Given, length of conductor, $l = 0.1\,\text{m}$

Mangetic field, $B = 0.1\,\text{T}$

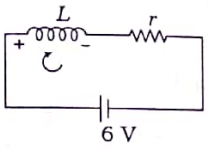
Velocity of conductor, $v = 15\,\text{ms}^{-1}$

The angle between v and B is 90°.

When v and B are mutually perpendicular, then emf (induced) is given by

$$\varepsilon = vBl = 15 \times 0.1 \times 0.1 = \frac{15}{100} = 0.15\,\text{V}$$

10. (*c*) Applying KVL in the circuit, we can write



$$6 - L\frac{dI}{dt} - Ir = 0$$

At initial stage, $I = 0$

$$\Rightarrow \quad 6 = L\frac{dI}{dt}$$

or $$\frac{dI}{dt} = \frac{6}{L} = \frac{6}{2} = 3\ \text{As}^{-1}$$

11. (*d*) As magnetic field through circle is constant because current through straight wire is constant. Magnetic flux through circle is not changing with time.

$$\frac{d\phi_B}{dt} = 0$$

$$\Rightarrow \quad |E| = \left|\frac{d\phi_B}{dt}\right| = 0 \Rightarrow i = \frac{E}{R} = \frac{0}{R} = 0$$

12. (*c*) We know that, $e = NBA\,\omega \sin \omega t$

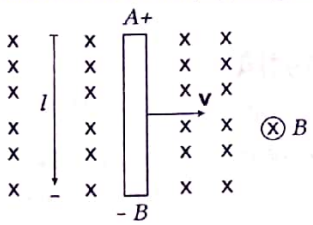
where, N = number of loops = 1

$$B = \frac{\mu_0 I}{2b}\ \text{N/A-m} \Rightarrow A = \pi a^2\ \text{m}^2$$

$$\therefore \quad e = \frac{\mu_0 I}{2b}(\pi a^2)\,\omega \sin \omega t$$

$$\Rightarrow \quad = \frac{\pi a^2 \mu_0 I}{2b} \cdot \omega \sin \omega t$$

13. (*c*) Consider the diagram, where a conductor AB of length l moving perpendicular to an inward magnetic field B.



Value of induced emf,

$$|E_{\text{ind}}| = |l(\mathbf{v} \times \mathbf{B})|$$

$$\Rightarrow \quad = lvB \sin 90° \qquad [\because v \perp B]$$

$$E_{\text{ind}} = vBl \qquad \ldots(i)$$

Given, $v = 7\ \text{ms}^{-1}$, $l = 0.4\ \text{m}$ and $B = 0.9\,\text{Wbm}^{-2}$

Now, from Eq. (i), we get

$$E_{\text{ind}} = (7\ \text{ms}^{-1})(0.9\ \text{Wbm}^{-2})(0.4\ \text{m})$$

$$= 7(0.9)(0.4)\ \text{V} = 2.52\ \text{V}$$

14. (*a*) Consider the inductor of inductance L.

The current flowing through the inductor is i.

Now, we can write $\phi = Li$

where, ϕ is magnetic flux linked with the inductor

$$\Rightarrow \quad \frac{d\phi}{dt} = L\frac{di}{dt}$$

Given, $L = 40\ \text{mH}$

dt = change in time $= t_2 - t_1 = 4\ \text{ms} = \Delta t$

$di = i_2 - i_1 = 11 - 1 = 10\ \text{A} = \Delta i$

So, $$\frac{d\phi}{dt} = \frac{\Delta\phi}{\Delta t} = L\frac{\Delta i}{\Delta t} = (40\ \text{mH})\frac{(10)}{4\ \text{ms}}$$

$$= 10 \times 10 = 100 \qquad \ldots(i)$$

According to Faraday's law of electromagnetic induction,

Emf induced, $$e = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad |e| = \frac{\Delta\phi}{\Delta t} = 100\ \text{V} \qquad \text{[from Eq. (i)]}$$

15. (*b*) $\because$ Induced emf, $e = -L\dfrac{di}{dt}$

Given, $L = 10\ \text{H}$, $\Delta i = 9 - 4 = 5\ \text{A}$, $dt = 0.2\ \text{s}$

emf, $$e = 10 \times \frac{5}{0.2} = 250\,\text{V}$$

16. (*c*) As, $e = Bvl$

Here, $B = 10^{-3}\ \text{Wb/m}^3$

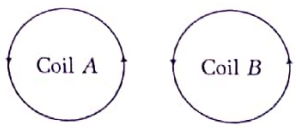
$v = 2\ \text{m/s}$

$l = 5\,\text{cm} = 0.05\,\text{m}$

$\therefore$ emf, $e = (10^{-3})\,(2)\,(0.05) = 1 \times 10^{-4}\ \text{V}$

17. (*c*) As the current in coil A changes, the magnetic flux linked with coil B changes. Direction of flux in B will be downwards, hence current induces in coil B is in anti-clockwise direction according to Lenz's law.

The direction of induced current in coil B is opposite to that of the direction of current in coil A as shown in figure



Due to currents in opposite directions in the near by sides of the coil, the coil B is repelled.

18. (*) Given, $n = 100$ turns, $A = 0.1 \times 0.05\ \text{m}^2$

$B_1 = 0.1\ \text{T}$, $B_2 = 0.05\ \text{T}$, $dt = 0.05\ \text{s}$

We know that,

$$e = \left|\frac{-d\phi}{dt}\right| = \left|\frac{d}{dt}(n\,B\,A\cos\theta)\right| = \left|\frac{nA\,dB\cos\theta}{dt}\right|$$

Here, $\theta = 0°$

$\therefore \quad e = \left|\frac{nAdB}{dt}\right| = \left|\frac{100 \times 0.1 \times 0.05 \times (0.1 - 0.05)}{0.05}\right|$

$\Rightarrow \quad e = 0.5$ V

None of the option is correct.

19. (*e*) Electromagnetic induction is not used in room heater because it works on the principle of heating effect of electric current.

20. (*d*) Given, $M = 0.05$ H and $I = I_0 \sin \omega t$

$\therefore \quad \frac{dI}{dt} = I_0 \cos \omega t(\omega)$

$\Rightarrow \left(\frac{dI}{dt}\right)_{max} = I_0(\omega) \times 1 = 1 \times 100\ \pi$ A/s

So, maximum emf, $e_{max} = M\left(\frac{dI}{dt}\right)_{max} = 0.05 \times 100\pi = 5\pi$ V

21. (*a*) The amount of induced charge is given by

$$q = \frac{1}{R}\Delta\phi = \frac{1}{100}(60 - 10)$$

[where, $\phi = 10$ Wb, $\phi_2 = 60$ Wb, $R = 100\,\Omega$]

$\therefore$ Charge, $\quad q = \frac{50}{100} = 0.5$ C

22. (*d*) For inductor as we know induced voltage

For $\quad t = 0$ to $t = T/2$

$$V = L\frac{dI}{dt} = L\frac{d}{dt}\left(\frac{2I_0 t}{T}\right) = \text{constant}$$

For $\quad t = T/2$ to $t = T$

$$V = \frac{LdI}{dt}\left(\frac{-2I_0 t}{T}\right) = -\text{constant}$$

Therefore, answer will represent by graph (d).

23. (*a*) Given, $B = 0.15$ T, $l = 50 \times 10^{-2}$ cm and $v = 2\,ms^{-1}$

$\therefore \quad$ emf, $e = Bvl$

$e = 0.15 \times 2 \times 50 \times 10^{-2} = 0.15$

Current, $i = \frac{e}{R} = \frac{0.15}{3} = 5 \times 10^{-2}$

Force, $F = Bil = 0.15 \times 5 \times 10^{-2} \times 50 \times 10^{-2}$

$= 3.75 \times 10^{-3}$ N

24. (*a*) Induced emf, $e = 2$ V

$i_1 = 8$ A, $i_2 = 6$ A

$\Delta t = 2 \times 10^{-3}$ s

Coefficient of self-induction,

$$L = \frac{e}{\Delta i/\Delta t} = \frac{-2}{(6-8)/2 \times 10^{-3}}$$

$$= \frac{-2 \times 2 \times 10^{-3}}{-2}$$

$$= 2 \times 10^{-3}\ \text{H}$$

25. (*c*) Rate of doing work $= \frac{W}{t} = P$

But, $\quad P = Fv$

Also, $\quad F = Bil = B\left(\frac{Bvl}{R}\right)l$

$\left[\because \text{induced current, } i = \frac{Bvl}{R}\right]$

$\therefore$ Power, $P = B\left(\frac{Bvl}{R}\right)lv = \frac{B^2v^2l^2}{R}$

$$= \frac{(0.5)^2 \times (2)^2 \times (1)^2}{6} = \frac{1}{6}\ \text{W}$$